Free Choice With Arbitrary Variables

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Abstract

This paper presents a uniform analysis of free choice constructions in English that incorporates a mechanism of arbitrary variability directly into their meaning. I propose that speakers interpret the values of certain variables or discourse referents as 'fungible', such that they could equally have taken any other value within an appropriate range. The semantics tracks this fungibility or arbitrariness, which can project to the sentential level and generate free choice readings with conjunctive or universal force.

1 Free choice: challenges to classicality

Free choice items and effects present sites of conflict between traditional logical semantics—i.e., the analysis of natural language in terms of classical concepts of modern logic—and the ways speakers naturally understand logical connectives, quantifiers, and modal expressions in everyday interpretation and inference. One one hand, free choice uses of *any* and *a* challenge a uniform treatment of 'universal force' in English in terms of the logician's $\forall$:

(1) Any owl is zygodactyl.

(2) A continuous function on a closed interval is bounded.

These uses essentially convey *arbitrariness*, indicating that a satisfying entity can be freely chosen from a domain of relevant items—owls, continuous functions, closed intervals, or whatnot—but this freedom of choice is not required and is often absent with other determiners:

(3) a. Any match {at all/whatsoever} that I strike lights {it doesn't matter which}.
b. {?Any/Every/Each} match is still in the box.

Relatedly, FC *any* and *a* statements are 'law-like'. They support counterfactual inferences (Ryle 1949; Vendler 1962) and do not carry existential commitments (second example from Horn 2000):

(4) {Any/An} owl is zygodactyl. So, if Tweety were an owl, Tweety would be zygodactyl.

(5) She may never marry, but {anyone/*everyone/*the person} she does marry will be Jewish.

Even more strikingly, in some examples FC *any* appears to have an existential meaning:

(6) Pick {any/?every} card.

This has prompted some linguists to argue that FC *any* is an indefinite that must acquire its universal force from quantificational elements external to the *any*-phrase (Kadmon and Landman 1990, 1993; Horn 2000; Giannakidou 2001).

Free choice uses of disjunction with 'conjunctive force', as in cases of "Free Choice Permission", present a further challenge to traditional logical semantics:

(7) You may have the whiskey or the gin. (Kamp 1974)

The standard Boolean account of disjunction combined with a Kripkean possible worlds semantics for modality fails to predict the free choice inference. This puzzle of FC disjunction extends to other modal flavors, *would* sentences, and examples where modality is not explicitly present:

(8) Mrs. X might be in Victoria or Brixton. (Zimmermann 2000)

(9) Chewy can fly the spaceship to Sirius or Hesperus. (Fusco 2021)

(10) I would dance with Mary or Sue. (Kadmon and Landman 1993)

(11) An apple or a pear costs a dollar.

Like the examples with FC *any* and *a*, such examples also convey arbitrariness with respect to the choice between disjuncts, which remains mysterious with a classical Boolean disjunction.

In the linguistics and philosophy literature, FC *any* and FC *or* are often studied independently. However, I agree with Chierchia (2013) that there are compelling reasons to accept what he calls the 'Identity Thesis': "free choice (FC) effects constitute a unitary phenomenon empirically, and call for a uniform explanation conceptually" (p. 50). First, universal generalizations generalize conjunctions (and existential indefinites generalize individual disjunctions), making it a natural working hypothesis that the universal force of FC *any* and *a* and the conjunctive force of FC *or* spring from a shared underlying source. Second, free choice uses of *any*, *a*, and *or* exhibit a strikingly similar distribution—they are readily available in modal environments, generally unavailable in episodic statements, and so on—supporting the pursuit of a unified analysis.

2 Proposal in brief: indetermination and arbitrariness

Existing accounts of free choice can be broadly categorized as either pragmatic or semantic. Among the pragmatic accounts, the most influential approaches seek to explain free choice within the system that derives Scalar Implicatures (SIs) (Fox 2007; Franke 2011; Chierchia, Fox, and Spector 2012; Chierchia 2013; Dayal 2013; Bar-Lev and Fox 2020). This assimilation of FC inferences to standard SIs is suggested by certain shared empirical properties, such as their disappearance under negation. However, while SI-based accounts are ingenious, they rely on particularly complex algorithms for calculating FC implicatures—for instance, Fox's (2007) original theory postulates a covert exhaustivity operator, EXH, which must be applied recursively to sets of alternatives, and subsequent accounts have done little to simplify this process, raising concerns about their cognitive plausibility, especially given the apparent ease with which speakers access free choice readings (see Chemla and Bott 2014; Tieu et al. 2016).

Semantic accounts of free choice items and effects necessitate departures from traditional logical semantics, and it is a testament to the creativity and ingenuity of linguists and philosophers just how many have been proposed (see Kadmon and Landman 1993; Dayal 1998; Zimmermann 2000; Simons 2005; Geurts 2005; Aloni 2007; Barker 2010; Aher 2012; Starr 2016; Goldstein 2019; Aloni 2022; among others). However, I find myself not entirely satisfied with any existing semantic accounts either. Some of the proposed systems have questionable logical properties, such as abandoning the Law of Non-Contradiction or failing to validate Dual Prohibition (as discussed by Goldstein 2019). A broader issue, though, is that the challenges posed by different free choice elements have often been approached in a piecemeal fashion, while I aim to uncover a principled and uniform conceptual basis for free choice effects in their entirety.

Given these discontents, I would like to pursue a different semantic strategy, one that takes to heart the essential *arbitrariness* or *freedom* inherent in free choice and, to my knowledge, has yet to be rigorously explored. Following a longstanding tradition, I analyze indefinites and individual disjunctions as free variables that stand for 'discourse referents', which are associated with an appropriate range of possible values (Karttunen 1969; Kamp 1981; Heim 1982). The core idea is to explicitly distinguish between two ways that speakers understand and handle these alternative values of variables or 'drefs'. In cases of 'mere indetermination', the individual identities of the alternative values are understood to *matter*, in the sense that some value or values may meet a condition in a sentence where the variable occurs, while others may not. For instance, if I say, 'An owl hooted', I convey that some particular owl hooted, leaving its exact identity unspecified. This existential interpretation can be derived by treating *an owl* as a variable ranging over an appropriate class of owls and handling it in a manner consistent with the idea that each alternative owl is a possible candidate for being the one owl that hooted.

However, speakers also introduce variables or drefs whose values are understood as entirely *arbitrary*; they could just as well have taken any other value within an appropriate range. When exhibiting this arbitrary variability, a variable is processed in a way that generates interpretations of generality, and this, I want to suggest, is what commonly underlies the phenomenon of free choice. While the free choice item *any owl* is also treated as a variable ranging over the owls, there is a crucial difference with non-FC uses of *an owl*: the value of *any owl* is understood to be an arbitrarily selected owl. I assume that the semantics keeps track of all other 'fungible values' that this dref could equally have taken. Free choice interpretations arise from the presence of this fungibility, which projects to the sentence level, generating conjunctive or universal force.

I myself arrived at this theory of free choice through reflecting on work by philosophical logicians on 'instantial reasoning', particularly on variable declarations like the following:[1]

(12) Let n be {a/any/an arbitrary} natural number.

In the same spirit as Fine (1983, 1985a,b), who interprets n as referring to an 'arbitrary number', and Breckenridge and Magidor (2012), who see n as 'arbitrarily referring' to a specific number, I understand (12) as introducing a new dref taking an arbitrary value from the natural numbers.

3 A variabilist semantics of choice

We can think of the value range of a variable as a 'menu' of options, with its taking a value as a 'choice' from these options (cf. Bledin 2024, drawing on Fine 2017a). In terms of this menu metaphor, the distinction I am drawing is between 'non-arbitrary' vs. 'arbitrary choice'.

I implement this distinction within a novel 'variabilist' semantics in which *all* phrases are analyzed as independent or dependent variables, whose values are recursively specified in terms of the values of any other variables, if any, on which they depend (in a slogan: 'to know the meaning of a variable is to know how its value is determined').[2] The value of a variable is either an *entity* or *state* (or 'eventuality'). The value range of a variable, or its menu, is thus a set of entities or a set of states (cf. compositional 'menu semantics' in Bledin 2024).

The value ranges of independent variables (names like *Ceres*, verbs like *sing*, and common nouns like *owl*) are specified by a valuation function V provided in a model for the fragment. The value of an independent v_{ind} is a choice from its range, represented using a nondeterministic function 'Choice' that returns a randomly selected element from a set if there are any to choose from; otherwise, Choice is undefined. Names are trivial variables, taking only a single entity value, while other independent variables can have value ranges with multiple options.

$$\begin{array}{lclcl} \textsc{Value}(\mathsf{Ceres}) & = & \text{Choice}(\text{V}(\mathsf{Ceres})) & = & \text{Choice}(\{\text{Ceres}\}) \\ \textsc{Value}(\mathsf{sing}) & = & \text{Choice}(\text{V}(\mathsf{sing})) & = & \text{Choice}(\{s : s \text{ is a singing}\}) \end{array}$$

Logically complex expressions like *Apollo or Ceres* or *sing and dance*, as well as full sentences like *Ceres sang*, are dependent variables whose values are derived from the values of their constituent variables. Disjunctive variables introduce further nondeterminism, where the value of such a variable is a choice between the values of its disjuncts (building on 'alternative semantics' (Hamblin 1973; Alonso-Ovalle 2006) and its descendants, such as 'truthmaker semantics' (Fine 2017a,b,c) and 'inquisitive semantics' (Ciardelli, Groenendijk, and Roelofsen 2013, 2018)).

[1] Russell's conception of variables in *The Principle of Mathematics* as denoting a special kind of "variable conjuntion" is another important historical precursor to the account.

[2] While my semantics makes heavy use of variables, it is worth noting that the treatment differs from the familiar Tarskian one inherited from first-order logic—while each variable takes values from a range of options, there is no environment of assignment functions mapping variables to their values.

$$\begin{array}{lll} \textsc{Value}(\textsf{Apollo or Ceres}) & = & \text{Choice}(\{\textsc{Value}(\textsf{Apollo}), \textsc{Value}(\textsf{Ceres})\}) \\ \textsc{Value}(\textsf{Apollo sang or Ceres danced}) & = & \text{Choice}(\{\textsc{Value}(\textsf{Apollo sang}), \textsc{Value}(\textsf{Ceres danced})\}) \end{array}$$

Conjunction has a 'collective' semantics, where the value of a conjunctive variable is the mereological fusion of the values of its conjuncts. I assume that the domains of entities and states are complete lattices, meaning that every set X of entities or states has a least upper bound, which I identify as the fusion, $\bigsqcup X$, of the members of this set ($x \sqcup y$ abbreviates $\bigsqcup\{x, y\}$). At the level of the determiner phrase, we have the Linkian analysis of individual conjunctions as plural entities (Link 1983; Hoeksema 1983; Krifka 1990; Lasersohn 1995; Schwarzschild 1996; Heycock and Zamparelli 2005; Schmitt 2013), while at the sentential level, we have the signature treatment of conjunction from truthmaker semantics (Fine 2017a,b,c).

$$\begin{array}{lll} \textsc{Value}(\textsf{Apollo and Ceres}) & = & \textsc{Value}(\textsf{Apollo}) \sqcup \textsc{Value}(\textsf{Ceres}) \\ \textsc{Value}(\textsf{Apollo sang and Ceres danced}) & = & \textsc{Value}(\textsf{Apollo sang}) \sqcup \textsc{Value}(\textsf{Ceres danced}) \end{array}$$

To determine the values of sentential variables, I adopt a Neo-Davidsonian account of argument structure on which states or eventualities are linked to their participants via thematic roles (Carlson 1984; Parsons 1990; Krifka 1992). For instance, $\text{Agent}(s) = \text{Ceres}$ indicates that the agent of the state s is Ceres. Thematic roles are introduced by syntactic correlates in LF, such as the silent theta-role heads [Ag] and [Th], which correspond to Agent and Theme (Kratzer 1996). (When I need to generalize, I use [TR] to designate an arbitrary theta-role head and TR to designate the corresponding arbitrary thematic role.)

$$\textsc{Value}(\textsf{[Ag]Ceres}) \quad = \quad \text{Choice}(\{s : \text{Agent(s)} = \textsc{Value}(\textsf{Ceres})\})$$

Additionally, I require some semantic glue: a concatenation operation (^) whereby the concatenation of two variables assumes a value only if that value is shared by both variables (cf. Predicate Modification, Pietroski's (2018) M-Junction).

$$\textsc{Value}(\textsf{[Ag]Ceres}^\wedge\textsf{sing}) \quad = \quad \textsc{Value}(\textsf{[Ag]Ceres}) \text{ if } \textsc{Value}(\textsf{[Ag]Ceres}) = \textsc{Value}(\textsf{sing})\text{; else Undef}$$

The core semantics of choice is summarized in the following definition:

Definition 1. The value of a variable v, $\textsc{Value}(v)$, is determined as follows:

$$\begin{array}{rcl} \textsc{Value}(v_{\text{ind}}) & = & \text{Choice}(\text{V}(v_{\text{ind}})) \\ \textsc{Value}(v \textsf{ or } u) & = & \text{Choice}(\{\textsc{Value}(v), \textsc{Value}(u)\}) \\ \textsc{Value}(v \textsf{ and } u) & = & \textsc{Value}(v) \sqcup \textsc{Value}(u) \text{ if neither } \textsc{Value}(v) \text{ nor } \textsc{Value}(u) \text{ is Undef; else Undef} \\ \textsc{Value}(\textsf{[TR]}\ v) & = & \text{Choice}(\{s : \text{TR}(s) = \textsc{Value}(v)\}) \\ \textsc{Value}(v^\wedge u) & = & \textsc{Value}(v) \text{ if } \textsc{Value}(v) = \textsc{Value}(u)\text{; else Undef} \end{array}$$

The determination of values in Def 1 induces the following structure of value ranges:

Definition 2. $\textsc{ValueRange}(v) \quad = \quad \{x : x \text{ is a possible determination of } \textsc{Value}(v)\text{, excluding Undef}\}$.

$$\begin{array}{rcl} \textsc{ValueRange}(v_{\text{ind}}) & = & \text{V}(v_{\text{ind}}) \\ \textsc{ValueRange}(v \textsf{ or } u) & = & \textsc{ValueRange}(v) \cup \textsc{ValueRange}(u) \\ \textsc{ValueRange}(v \textsf{ and } u) & = & \{x \sqcup y : x \in \textsc{ValueRange}(v), y \in \textsc{ValueRange}(u)\} \\ \textsc{ValueRange}(\textsf{[TR]}\ v) & = & \{s : \text{TR}(s) \in \textsc{ValueRange}(v)\} \\ \textsc{ValueRange}(v^\wedge u) & = & \textsc{ValueRange}(v) \cap \textsc{ValueRange}(u) \end{array}$$

At this point, we effectively have a cross-categorial unilateral version of truthmaker semantics, where the state values of sentential variables can be regarded as their truthmakers:

Definition 3. The proposition $[\![v_{\text{sent}}]\!]$ expressed by a sentence v_{sent}, or its 'truthmaker content', is $\textsc{ValueRange}(v_{\text{sent}})$ (to be revised below).

(13) $$\begin{array}{ll} & \textsc{ValueRange}(\textsf{[[Ag]Ceres]}^\wedge\textsf{[sing or dance]}) \\ = & \{s : \text{Agent}(s) = \text{Ceres} \wedge (s \in \text{V}(\textsf{sing}) \vee s \in \text{V}(\textsf{dance}))\} \end{array}$$

In (13), a truthmaker for *Ceres sang or danced* is a state of Ceres singing or one of her dancing.

4 Introducing arbitrary choice

Let us now introduce the possibility of arbitrary choice by associating a variable with 'fungible values', the values it could just as well have taken:

Definition 4. $\textsc{Value}^*(v) \;=\; \langle\textsc{Value}(v), \textsc{FungibleValues}(v)\rangle$
where $\textsc{Value}(v) \in \textsc{FungibleValues}(v)$

The following upgrade to Def 1 details how fungible values project in the natural way:

Definition 5. $\textsc{Value}^*(v)$ is determined as follows:

$$\begin{array}{rcl}\textsc{Value}^*(v_{\text{ind}}) & = & \langle\text{Choice}(\text{V}(v_{\text{ind}})), \{\text{Choice}(\text{V}(v_{\text{ind}}))\}\rangle \\ \textsc{Value}^*(v \text{ or } u) & = & \text{Choice}(\{\textsc{Value}^*(v), \textsc{Value}^*(u)\}) \\ \textsc{Value}^*(v \text{ and } u) & = & \langle\textsc{Value}(v) \sqcup \textsc{Value}(u) \text{ if } ..., \\ & & \{x \sqcup y \text{ if } ... : x \in \textsc{FungibleValues}(v), y \in \textsc{FungibleValues}(v)\}\rangle \\ & & ... \text{ similarly for [TR] } v \text{ and } v^\wedge u.\end{array}$$

We have no real fungibility yet: $\textsc{Value}^*(v) = \langle\textsc{Value}(v), \{\textsc{Value}(v)\}\rangle$ for all v. But I assume an arbitrariness operator, ARB, can apply to a nondeterministic Choice-y variable v to yield a new variable, ARB(v), which is just like v except its fungible values include all those that could have been randomly selected by the Choice function, taking into account that variables on which v depends—and thus the set to which Choice is applied—may involve some arbitrariness.

Definition 6. When $\textsc{Value}(v) = \text{Choice}(S(\textsc{Value}(v_1), \textsc{Value}(v_2), ...))$:

$$\begin{array}{rcl}\textsc{Value}^*(\text{ARB}(v)) & = & \langle\textsc{Value}(v), \bigcup\{S(x_1, x_2, ...) : x_1 \in \textsc{FungibleValues}(v_1), \\ & & x_2 \in \textsc{FungibleValues}(v_2), ...\}\rangle\end{array}$$

For example, we can now represent an arbitrary choice between two individuals as follows:

(14) $\textsc{Value}^*(\text{ARB(Mary or Sue)}) \;=\; \langle\text{Choice}(\{\text{Mary}, \text{Sue}\}), \{\text{Mary}, \text{Sue}\}\rangle$

5 FC *or*

To derive FC disjunction inferences via fungibility projection, I revise the notion of truthmaker content as follows, and appeal to the following notion of consequence (though other consequence relations are available within the truthmaker framework that would work here).

Definition 7. The 'truthmaker content' of a sentential variable v_{sent} is

$$[\![v_{\text{sent}}]\!] \;=\; \{\bigsqcup X : X \text{ is a possible determination of } \textsc{FungibleValues}(v_{\text{sent}}), \text{ excluding any X that includes Undef}\}$$

Definition 8. $v_{\text{sent}} \models u_{\text{sent}}$ iff each verifier $s \in [\![v_{\text{sent}}]\!]$ contains some verifier $t \in [\![u_{\text{sent}}]\!]$ and each $t \in [\![u_{\text{sent}}]\!]$ is contained in some $s \in [\![v_{\text{sent}}]\!]$ ("conjunctive parthood"; Fine 2017a,b,c; Yablo 2014).

In the following example, I assume the FC disjunction *Mary OR Sue* is interpreted as in (14):

(15) a. Mary OR Sue is a good choice for department chair.
b. $[\![$[[Pos]ARB(Mary or Sue)]$^\wedge$[is a good choice for department chair]$]\!]$
$= \{s \sqcup t : \text{Possessor}(s) = \text{Mary} \wedge \text{Possessor}(t) = \text{Sue} \wedge s, t \in \textsc{ValueRange}(\text{is good...})\}$

Clearly (15-a) $\models$ *Mary is a good choice and so is Sue.* Despite being computed in different ways, these sentences have the same truthmaker content (cf. Frege on *Sinn* and *Bedeutung*).

Free choice permission statements are treated similarly. Without committing to a specific theory of modality, I rely on the following minimal assumptions: (i) like non-modal sentences, permission statements have exact verifiers—possibly special 'modal states' (Fine in Hale 2020; Güngör 2024); and (ii) $\textsc{Value}$(might/may v_{sent}) is a function of $\textsc{Value}(v_{\text{sent}})$. My notation: if $\textsc{Value}(v_{\text{sent}}) = s$ then $\textsc{Value}$(might/may v_{sent}) $= s_\Diamond$.

(16) a. You may have whiskey or gin.
b. $[\![\text{may}[[\text{Ag}]\text{you}]^\wedge\text{have}^\wedge[[\text{Th}]\text{ARB}(\text{whiskey or gin})]]]\!]$
$=$ $\{s_\Diamond \sqcup t_\Diamond : s \in \text{VALUERANGE}(\text{you have whiskey}), t \in \text{VALUERANGE}(\text{you have gin})\}$

(16-a) $\models$ *You may have whiskey and you may have gin.* Both (16-a) and the conjunction share the same truthmaker content as the sentential disjunction *You may have whiskey OR you may have gin*, therefore the semantics also accommodates Wide Scope Free Choice.

6 FC *any*

The treatment of FC *any* and *a* largely parallels that of FC *or*.

Definition 9. $\text{VALUE}(\mathsf{a}\ v) = \text{Choice}(\{e : \exists s \in \text{VALUERANGE}(v)(\text{Possessor}(s) = e)\})$ (ignoring domain restriction; see Bledin 2024 for relevant discussion; see also 'choice function' approaches to indefinites by Reinhart 1997; Winter 1997; Kratzer 1998; Matthewson 1998))

(17) $\text{VALUE}^*(\mathsf{ARB}(\text{an owl})) = \langle \text{Choice}(\{\text{owl1}, \text{owl2}, ...\}), \{\text{owl1}, \text{owl2}, ...\}\rangle$

I assume that while ARB is optional with *a*, it is obligatory with *any*:

(18) a. Any owl is zygodactyl.
b. $[\![[[\text{Pos}]\text{ARB}(\text{an owl})]^\wedge[\text{is zygodactyl}]]\!]$
$=$ $\{s_1 \sqcup s_2 \sqcup ... : \text{Possessor}(s_1) = \text{owl1} \wedge s_1 \in \text{V}(\text{zygodactyl}) \wedge ...\}$

This captures many of the distinctive meaning facts about FC *any*: (i) its *free choiceness* is built into the very use of arbitrary variables; (ii) its universal force arises from the projection of fungibility and the definition of truthmaker content; (iii) since FC *any* is an indefinite, it does not require the existential commitment associated with universal determiners like *every*; and (iv) although FC *any* isn't itself modal, the use of arbitrary variables naturally gives rise to law-like 'essentializing' inferences, or property dependencies, which in turn support counterfactuals. By deploying an arbitrary variable with ARB, a speaker deemphasizes the individual identities of the values in its range and foregrounds their common properties. We can think of ARB(*an owl*) as functioning, so to speak, like a peg on which to hang properties shared by owls. When a speaker predicates being zygodactyl, this has the effect of adding this new property to the peg.

7 Loose ends and extensions

The main outstanding issue with my proposal is the *problem of distribution*: I need to account for restrictions on ARB and the limited availability of FC readings in episodic statements and other contexts. Multiple factors may be at play—blocking effects, lexical restrictions, and the sheer implausibility of FC interpretations in certain environments.

I am currently exploring ways to incorporate negation into the system by enabling the semantics to track 'excluded values', the values a variable cannot take.

Definition 10. $\text{VALUE}^{**}(v) = \langle \text{VALUE}(v), \text{FUNGIBLEVALUES}(v), \text{EXCLUDEDVALUES}(v)\rangle$

I believe this has many potential applications beyond free choice, from hard cases involving the integration of negation with collective conjunction (cf. Bledin 2024), to modified numerals (particularly "van Benthem's puzzle"; Krifka 1999; Brasoveanu 2013), and more.

Another potential application is accounting for the regular vs. "exceptional" scope of indefinites in conditionals, not through scope-taking (see Charlow 2020 for a recent proposal), but by appealing to the arbitrary-nonarbitrary distinction:

(19) If a rich relative of mine dies, I'll inherit a house.

The different readings of this conditional may depend on whether *a rich relative of mine* is treated as an arbitrary or nonarbitrary variable.

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