Graduate Seminar: Truthmakers

Johns Hopkins University, Fall 2020

Course Information

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Office Hours	by appt only
Class Code	AS.150.625
Class Time	F 10:00am-12:00pm EST
Zoom ID	995 6820 7623 (Password: 998525)

Course Description

An investigation into the theory of truthmaker semantics and its applications to various problems in philosophy and linguistics.

Requirements

One paper on truthmakers, roughly thirty pages double-spaced. If you are a graduate student taking this seminar to fulfill the logic requirement, then your paper must have a strong technical component.

Schedule

I want to keep the seminar open-ended and decide on readings as we go. That said, we'll begin by working through Kit Fine's recent survey article on truthmaker semantics in the Blackwell philosophy of language volume to get an overview of the seminar, and then read some early papers on truthmaking by Bas van Fraassen and others. Later in the semester, I also want to devote at least a couple of weeks to linguistic applications of truthmaker semantics, especially in recent work by Friederike Moltmann.

Here are some things we could read. We almost certainly won't cover all of this material and I expect to add more readings as we delve deeper into various subject matters.

Historical background

Bas van Fraassen. Facts and Tautological Entailments. *Journal of Philosophy* 66(15):477–487, 1969.

Kevin Mulligan, Peter Simons, and Barry Smith. Truth-Makers. *Philosophy and Phenomenological Research* 44(3):287–321, 1984.

Truthmaker semantics

Kit Fine. Truthmaker Semantics for Intuitionistic Logic. *Journal of Philosophical Logic*, 43(2):

549–577, 2014.

Kit Fine. Angelic Content. *Journal of Philosophical Logic*, 45(2): 199–226, 2016.

Kit Fine. Truth-Maker Content I: Conjunction, Disjunction and Negation. *Journal of Philosophical Logic*, 46(6): 625–674, 2017.

Kit Fine. Truth-Maker Content II: Subject Matter, Common Content, Remainder and Ground. *Journal of Philosophical Logic*, 46(6): 675–702, 2017.

Kit Fine. Truthmaker Semantics. In Bob Hale, Alexander Miller, and Crispin Wright, editors, *A Companion to the Philosophy of Language*, pages 556-577. Blackwell, 2017.

Kit Fine. Yablo on subject-matter. *Philosophical Studies* 177(1):129–171, 2020.

Kit Fine and Mark Jago. *An Introduction to Truthmaker Semantics.* Book manuscript.

Stephen Yablo. *Aboutness.* Princeton University Press, Princeton, 2014.

Stephen Yablo. Precis of Aboutness. *Philosophical Studies* 174(3):771–777, 2017.

Conditionals

Kit Fine. Counterfactuals without Possible Worlds. *Journal of Philosophy*, 109(3):221–246, 2012.

Paolo Santorio. Alternatives and Truthmakers in Conditional Semantics. *Journal of Philosophy* 115(10): 513–549, 2018.

Stephen Yablo. Ifs, Ands, and Buts: An Incremental Truthmaker Semantics for Indicative Conditionals. *Analytic Philosophy* 57(1): 175–213, 2016.

Imperatives and deontic logic

Kit Fine. Compliance and Command I - Categorical Imperatives. *Review of Symbolic Logic*, 11(4): 609–633, 2018.

Kit Fine. Compliance and Command II - Imperatives and Deontics. *Review of Symbolic Logic*, 11(4): 634–664, 2018.

Daniel Rothschild and Stephen Yablo. Permissive Updates. Forthcoming in a volume for Kit Fine.

More linguistic applications

Friederike Moltmann. Truthmaker Semantics for Natural Language: Attitude Verbs, Modals, and Intensional Transitive Verbs. To appear in *Theoretical Linguistics.*

Friederike Moltmann. Situations, Alternatives, and the Semantics of 'Cases'. *Linguistics and Philosophy*, 2019.

Related approaches

Jon Barwise and John Perry. Situations and Attitudes. Cambridge, MIT Press, 1983.

Angelika Kratzer. Situations in Natural Language Semantics. Stanford Encyclopedia of Philosophy.

Michael Deigan. A Plea for Inexact Truthmaking. *Linguistics and Philosophy*, 2019.

Ivano Ciardelli, Jeroen Groenendijk, and Floris Roelofsen. *Inquisitive Semantics.* Oxford University Press, Oxford, 2019.

Truthmakers, Total and Partial

AS.150.645: Truthmakers
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1 Total Truthmakers: Possible worlds

While possible worlds have been taken to earn their keep in semantics for modal languages, let's consider a simple non-modal sentential language:

Definition 1. The **basic language** $\mathcal{L}$ has the following generative syntax (in *Backus-Naur* notation):

$p ::= \mathsf{A}, \mathsf{B}, \mathsf{C}, ...$
$\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi)$

Let $At_{\mathcal{L}} = \{\mathsf{A}, \mathsf{B}, ...\}$ be the set of atoms in $\mathcal{L}$, and $S_{\mathcal{L}}$ be the set of all well-formed sentences in $\mathcal{L}$. We can define the material conditional '$\supset$' and biconditional '$\equiv$' in the usual fashion.

For expository purposes, I'll simply identify possible worlds with truth value assignments to all sentence letters in $At_{\mathcal{L}}$ (rather than taking them to be primitive points in a model at which sentence letters are evaluated by a separate valuation function). I'll also work with a single all-encompassing model that includes all the possible worlds:

Definition 2. The **worldly model** $\mathcal{M} = \langle \mathcal{W} \rangle$ for $\mathcal{L}$ consists of the set $\mathcal{W} = \{T, F\}^{At_{\mathcal{L}}}$ of all **possible worlds**, where each world $w \in \mathcal{W}$ is a total function mapping each sentence letter $p \in At_{\mathcal{L}}$ to a truth value.

Definition 3. Truth at a world, $w \models \varphi$, is recursively defined for each sentence $\varphi \in S_{\mathcal{L}}$ in the language as follows:

$$\begin{array}{lll} w \models p & \text{iff} & w(p) = T \\ w \models \neg\varphi & \text{iff} & w \not\models \varphi \\ w \models \varphi \wedge \psi & \text{iff} & w \models \varphi \text{ and } w \models \psi \\ w \models \varphi \vee \psi & \text{iff} & w \models \varphi \text{ or } w \models \psi \end{array}$$

We can define the usual constellation of logical notions in terms of truth at a world:

Definition 4. The argument from premises $\varphi_1, ..., \varphi_n$ to conclusion ψ is **logically valid**, $\{\varphi_1, ..., \varphi_n\} \models \psi$, iff there is no world $w \in \mathcal{W}$ such that $w \models \varphi_1$, $w \models \varphi_2$, ... but $w \not\models \psi$.

Definition 5. The sentence φ is a **logical validity**, $\models \varphi$, iff there is no world $w \in \mathcal{W}$ such that $\mathcal{M}, w \not\models \varphi$.

Definition 6. The sentences $\varphi_1, ..., \varphi_n$ are **logically consistent** iff there is some world $w \in \mathcal{W}$ such that $w \models \varphi_1$, ..., and $w \models \varphi_n$.

And so forth.

Having defined truth at a world, we can associate a *proposition* with each sentence $\varphi \in S_{\mathcal{L}}$: $[\varphi] = \{w \in \mathcal{W} : w \models \varphi\}$ (i.e., $[\varphi]$ is the set of possible worlds where φ is true). Equivalently, we can state our semantic clauses in terms of propositions and take truth at a world to be the derived notion:

Definition 7. The **propositional content** $[\varphi]$ of $\varphi \in S_{\mathcal{L}}$ is recursively defined as follows:

$$\begin{array}{rcl} [p] & = & \{w \in \mathcal{W} : w(p) = T\} \\ [\neg\varphi] & = & \mathcal{W} \setminus [\varphi] \\ [\varphi \wedge \psi] & = & [\varphi] \cap [\psi] \\ [\varphi \vee \psi] & = & [\varphi] \cup [\psi] \end{array}$$

We then have $w \models \varphi$ iff $w \in [\varphi]$. Logical validity and friends can also be defined directly in terms of propositions; for instance:

Definition 8. $\{\varphi_1, ..., \varphi_n\} \models \psi$ iff $\bigcap\{[\varphi_1], ..., [\varphi_n]\} \subseteq [\psi]$.

Definition 9. $\models \varphi$ iff $[\varphi] = \mathcal{W}$.

2 States

Evaluating sentences with respect to entire possible worlds (i.e., maximally specific possibilities, determinate in every respect) might seem like overkill. After all, the truth or falsity of many sentences depends on the goings-on in small confined regions of reality—for example, the truth of *It's raining in Baltimore* doesn't turn on the weather in Amsterdam.

To capture this fineness of grain, we can evaluate sentences relative to partial rather than total functions:

Definition 10. The **stately model** $\mathcal{M} = \langle \mathcal{S} \rangle$ for $\mathcal{L}$ consists of the set $\mathcal{S} = \text{Pfn}(At_{\mathcal{L}}, \{T, F\})$ of all **states**, where each state $s \in \mathcal{S}$ is a partial function mapping sentence letters in $\text{dom}(s) \subseteq At_{\mathcal{L}}$ to truth values.

Note that while an atomic sentence can be neither true nor false in a state, we aren't yet allowing for *impossible states* in which a sentence can be both true and false.

We can define a mereology on $\mathcal{S}$ as follows:

Definition 11. s is **part** of s', $s \leqslant s'$, iff for all $p \in \text{dom}(s)$, $s(p) = s'(p)$.

This parthood relation is clearly a partial order (reflexive, anti-symmetric, and transitive).

Definition 12. The **fusion** of s and s', $s \oplus s'$, is defined only if for all $p \in \text{dom}(s) \cap \text{dom}(s')$, $s(p) = s'(p)$. When defined, $s \oplus s' = s \cup s'$.

Note that $\mathcal{S}$ contains a *null state* which is undefined everywhere and is therefore part of every other state:

Definition 13. The **null state** $s_\Box \in \mathcal{S}$ has $\text{dom}(s_\Box) = \emptyset$.

Directly above the null state are *atomic states*, viz. those states in $\mathcal{S}$ for which $\text{dom}(s)$ is a singleton. These states are the *exact verifiers* and *exact falsifiers* for atomic sentences in $At_\mathcal{L}$:

Definition 14. The state $s^+_p \in \mathcal{S}$ such that $s^+_p(p) = T$ and $s^+_p(q)$ is undefined for $q \neq p$ **exactly verifies** p.

Definition 15. The state $s^-_p \in \mathcal{S}$ such that $s^-_p(p) = F$ and $s^-_p(q)$ is undefined for $q \neq p$ **exactly falsifies** p.

We can regard all of the non-null states in $\mathcal{S}$ as built up from these atomic states. For example, the state s such that $s(\mathsf{A}) = T$, $s(\mathsf{B}) = F$, $s(\mathsf{C}) = T$, and $s(p)$ is otherwise undefined is the fusion $s^+_\mathsf{A} \oplus s^-_\mathsf{B} \oplus s^+_\mathsf{C}$. The highest (maximal) states in this mereology are the possible worlds $\mathcal{W} \subset \mathcal{S}$.

How to specify truth and falsity (or verification and falsification) conditions for the full language $\mathcal{L}$ within this state-based model theory? There are a couple of options. The first involves "inexact" verification/falsification:

Definition 16. Inexact verification and falsification by a state, $s \models_I \varphi$ and $s \mathrel{=\!\!|}_I \varphi$, is defined recursively as follows:

$$
\begin{array}{lll}
s \models_I p & \text{iff} & s(p) = T \text{ (i.e., } s^+_p \leqslant s) \\
s \mathrel{=\!\!|}_I p & \text{iff} & s(p) = F \text{ (i.e., } s^-_p \leqslant s) \\
s \models_I \neg\varphi & \text{iff} & s \mathrel{=\!\!|}_I \varphi \\
s \mathrel{=\!\!|}_I \neg\varphi & \text{iff} & s \models_I \varphi \\
s \models_I \varphi \wedge \psi & \text{iff} & s \models_I \varphi \text{ and } s \models_I \psi \\
s \mathrel{=\!\!|}_I \varphi \wedge \psi & \text{iff} & s \mathrel{=\!\!|}_I \varphi \text{ or } s \mathrel{=\!\!|}_I \psi \\
s \models_I \varphi \vee \psi & \text{iff} & s \models_I \varphi \text{ or } s \models_I \psi \\
s \mathrel{=\!\!|}_I \varphi \vee \psi & \text{iff} & s \mathrel{=\!\!|}_I \varphi \text{ and } s \mathrel{=\!\!|}_I \psi
\end{array}
$$

A simultaneous induction on the complexity of formulae establishes the following *persistence* (aka. *hereditary*) properties:

Fact 1. For any $\varphi \in S_\mathcal{L}$, if $s \models_I \varphi$ and $s \leqslant s'$ then $s' \models_I \varphi$.

Fact 2. For any $\varphi \in S_\mathcal{L}$, if $s \mathrel{=\!\!|}_I \varphi$ and $s \leqslant s'$ then $s' \mathrel{=\!\!|}_I \varphi$.

The second state semantics is based on "exact" verification/falsification (Fine 2017):

Definition 17. Exact verification and falsification by a state, $s \models_E \varphi$ and $s \mathrel{=\!\!|}_E \varphi$, is defined recursively as follows:

$$
\begin{array}{lll}
s \models_E p & \text{iff} & s = s^+_p \\
s \mathrel{=\!\!|}_E p & \text{iff} & s = s^-_p \\
s \models_E \neg\varphi & \text{iff} & s \mathrel{=\!\!|}_E \varphi \\
s \mathrel{=\!\!|}_E \neg\varphi & \text{iff} & s \models_E \varphi \\
s \models_E \varphi \wedge \psi & \text{iff} & \exists t \exists u (s = t \oplus u \text{ and } t \models_E \varphi \text{ and } u \models_E \psi) \\
s \mathrel{=\!\!|}_E \varphi \wedge \psi & \text{iff} & s \mathrel{=\!\!|}_E \varphi \text{ or } s \mathrel{=\!\!|}_E \psi \\
s \models_E \varphi \vee \psi & \text{iff} & s \models_E \varphi \text{ or } s \models_E \psi \\
s \mathrel{=\!\!|}_E \varphi \vee \psi & \text{iff} & \exists t \exists u (s = t \oplus u \text{ and } t \mathrel{=\!\!|}_E \varphi \text{ and } u \mathrel{=\!\!|}_E \psi)
\end{array}
$$

In this context, persistence can easily fail:

Fact 3. $s^+_\mathsf{A} \models_E \mathsf{A}$ but $s^+_\mathsf{A} \oplus s^+_\mathsf{B} \not\models_E \mathsf{A}$.

Fact 4. $s^-_\mathsf{A} \mathrel{=\!\!|}_E \mathsf{A}$ but $s^-_\mathsf{A} \oplus s^-_\mathsf{B} \not\mathrel{=\!\!|}_E \mathsf{A}$.

How are exact and inexact settlement related to one another? Another simple induction on formulae complexity establishes that a state inexactly verifies/falsifies φ iff it contains an exact verifier/falsifier for φ:

Fact 5. $s \models_I \varphi$ iff for some state $s' \leqslant s$, $s' \models_E \varphi$.

Fact 6. $s \mathrel{=\!\!|}_I \varphi$ iff for some state $s' \leqslant s$, $s' \mathrel{=\!\!|}_E \varphi$.

In the other direction, it's tempting to think that an exact verifier/falsifier for φ is simply a minimal (smallest) inexact verifier/falsifier (Fine 2017 calls this "minimalitis"):

- $s \models_E \varphi$ iff $s \models_I \varphi$ and there is no $s' < s$ s.t. $s' \models_I \varphi$.
- $s \mathrel{=\!\!|}_E \varphi$ iff $s \mathrel{=\!\!|}_I \varphi$ and there is no $s' < s$ s.t. $s' \mathrel{=\!\!|}_I \varphi$.

However, while these conditions hold for the special case where φ is atomic, they don't generalize to all sentences (Fine 2017). Consider $\mathsf{A} \vee (\mathsf{A} \wedge \mathsf{B})$. The exact verifiers for this sentence are s^+_A and $s^+_\mathsf{A} \oplus s^+_\mathsf{B}$ but the latter isn't a minimal verifier.

Interestingly, we can define many different consequence relations in terms of exact and/or inexact verification and/or falsification. One prima facie natural option is to have validity preserve inexact verification:

Definition 18. $\{\varphi_1, ..., \varphi_n\} \models_{S1} \psi$ iff there is no state $s \in \mathcal{S}$ such that $s \models_I \varphi_1$, $s \models_I \varphi_2$, ... but $s \not\models_I \psi$.

It can be shown by induction that for any $\varphi \in S_\mathcal{L}$, $w \models_I \varphi$ iff $w \models \varphi$, and $w \mathrel{=\!\!|}_I \varphi$ iff $w \not\models \varphi$, so we have:

Lemma 1. If $\{\varphi_1, ..., \varphi_n\} \models_{S1} \psi$ then $\{\varphi_1, ..., \varphi_n\} \models \psi$.

But the converse doesn't hold. Note in particular that $\models \mathsf{A} \vee \neg\mathsf{A}$ but

Fact 7. $\not\models_{S1} \mathsf{A} \vee \neg\mathsf{A}$. (as $s(\mathsf{A})$ can be undefined for some $s \in \mathcal{S}$)

To get classical logic, we must require that any state that inexactly verifies each of the premises doesn't inexactly falsify the conclusion:

Definition 19. $\{\varphi_1, ..., \varphi_n\} \models_{S2} \psi$ iff there is no state $s \in \mathcal{S}$ such that $s \models_I \varphi_1$, $s \models_I \varphi_2$, ... but $s \mathrel{=\!\!|}_I \psi$.

Lemma 2. $\{\varphi_1, ..., \varphi_n\} \models_{S2} \psi$ iff $\{\varphi_1, ..., \varphi_n\} \models \psi$.

One might also wonder about a consequence relation that preserves exact verification:

Definition 20. $\{\varphi_1, ..., \varphi_n\} \models_{S3} \psi$ iff there is no state $s \in \mathcal{S}$ such that $s \models_E \varphi_1$, $s \models_E \varphi_2$, ... but $s \not\models_E \psi$.

But this results in a very weak logic. We don't even get conjunction elimination:

Fact 8. $\{\mathsf{A} \wedge \mathsf{B}\} \not\models_{S3} \mathsf{A}$.

There are more interesting options within the exact truthmaker semantics framework. The following consequence relation is an attempt to capture *containment*, i.e., when the truth of the conclusion is contained in the truth of the premises:

Definition 21. $\{\varphi_1, ..., \varphi_n\} \models_{S4} \psi$ iff the following conditions both hold:

- For every set of states $s_1, ..., s_n$ such that $s_1 \models_E \varphi_1$, ..., $s_n \models_E \varphi_n$, and $s_1 \oplus ... \oplus s_n$ is defined, there exists a state $t \leqslant s_1 \oplus ... \oplus s_n$ such that $t \models_E \psi$.
- For every state t such that $t \models_E \psi$, there exist states $s_1, ..., s_n$ such that $s_1 \models_E \varphi_1$, ..., $s_n \models_E \varphi_n$, and $t \leqslant s_1 \oplus ... \oplus s_n$.

Some facts about this:

Fact 9. $\{\mathsf{A} \wedge \mathsf{B}\} \models_{S4} \mathsf{A}$.

Fact 10. $\{\mathsf{A}\} \not\models_{S4} \mathsf{A} \vee \mathsf{B}$. (second condition is violated)

Fact 11. $\{\mathsf{A} \vee \mathsf{B}\} \not\models_{S4} \mathsf{A}$. (first condition is violated)

Fact 12. $\{\mathsf{A} \vee \mathsf{B}, \neg\mathsf{A}\} \models_{S4} \mathsf{B}$.

Potential applications of this kind of containment/parthood consequence relation (from Yablo 2014):

- *Partial truth: Snow is white and expensive* is made partly true by the truth of *Snow is white. Snow is white* isn't made partly true by the truth of *Snow is white or expensive.*
- *Saying:* Someone who says that snow is white and expensive has said that snow is white, but someone who says that snow is white hasn't said that snow is white or expensive.
- *Musts and Mights:* Ordering someone to eat pork chops is ordering them to eat pork, but ordering someone to eat pork isn't ordering them to eat pork or human flesh. Likewise, if I might have pork chops for dinner then I might have pork, but if I might have pork for dinner then this doesn't entail that I might have pork or human flesh.
- *Confirmation:* The claim that all ravens are black is better confirmed by the evidence that a particular raven is black than by the evidence that all ravens are black, or they're all white, or they're all red, and so forth.

Fine-style Truthmaking

AS.150.645: Truthmakers
Johns Hopkins University, Fall 2020

1 State spaces

Definition 1. A **partially ordered set** (or **poset**) $\langle S, \leqslant \rangle$ is a set S with a reflexive, antisymmetric and transitive relation $\leqslant$ on it.

In the rest of this note, we'll assume that S is a set of *states* and $\leqslant$ is a *parthood* relation. Importantly, Fine (2017) regards 'state' as a technical term that needn't be interpreted in any intuitive sense:

> It is also important in applying the semantics to appreciate that the term 'state' is a mere term of art and need not be a state in any intuitive sense of the term. Thus facts or events or even ordinary individuals could, in principle, be taken to be states, as long as they are capable of being endowed with the relevant mereological structure and can be properly regarded as verifiers. (p. 560)

Some relevant definitions for posets:

Definition 2. Given a subset of states $T \subseteq S$,

- s is an **upper bound** of T iff $t \leqslant s$ for each $t \in T$.
- s is a **lower bound** of T iff $s \leqslant t$ for each $t \in T$.
- s is a **least upper bound/supremum/join** of T iff s is an upper bound of T and $s \leqslant s'$ for any upper bound s' of T.
- s is a **greatest lower bound/infimum/meet** of T iff s is a lower bound of T and $s' \leqslant s$ for any lower bound s' of T.

Definition 3. A poset $\langle S, \leqslant \rangle$ is **complete** iff every subset $T \subseteq S$ has a least upper bound $\bigoplus T \in S$.

Definition 4. Given a subset of states $T \subseteq S$,

- The **downward closure** of T is $T\downarrow = \{s : s \leqslant t \text{ for some } t \in T\}$.
- The **upward closure** of T is $T\uparrow = \{s : t \leqslant s \text{ for some } t \in T\}$.
- T is **downward closed** iff $T = T\downarrow$.
- T is **upward closed** iff $T = T\uparrow$.

Definition 5. A **state space** $\langle S, \leqslant \rangle$ is a complete poset where the set of states S is nonempty.

Given completeness, a state space includes a **full state** $\bigoplus S$ (henceforth $\blacksquare$) that fuses together all states and of which every state is thus a part, and a **null state** $\bigoplus \emptyset$ (henceforth $\square$) that is the fusion of no states and is therefore part of every state.

When the distinction between possible and impossible states is important, we can work with *modalized* state spaces, which have both mereological and modal structure:

Definition 6. A **modalized state space** $\langle S, S^\Diamond, \leqslant \rangle$ consists of a state space $\langle S, \leqslant \rangle$ and a nonempty subset of **possible states** $S^\Diamond \subseteq S$ that is downward closed.

Mereological and modal structure interact in the following definitions:

Definition 7. States $s, t \in S$ are **compatible** if $s \oplus t \in S^\Diamond$; otherwise, they are **incompatible**.

Definition 8. State $s \in S$ is a **world-state** if $s \in S^\Diamond$ and either $t \leqslant s$ or $s \oplus t \notin S^\Diamond$ for each $t \in S$.

Definition 9. A **W-space** is a modalized state space $\langle S, S^\Diamond, \leqslant \rangle$ such that every possible state $s \in S^\Diamond$ is part of a world-state.

While possible worlds are thus recoverable within certain state spaces, Fine is still a downer on possible worlds:

> Thus a W-space will, in effect, contain the pluriverse of possible worlds. However, very few applications require the assuption that the state space be a W-space and so, from this perspective, the postulation of possible worlds is a gratuitous assumption that serves no real purpose. (p. 561)

Brief comparison of Finean state spaces and the "stately model" from last time:

- While we earlier identified states with partial functions from atomic sentences to truth values, Fine wants to remain almost completely noncommittal about what states are (see above quote). In his highly abstract setting, states are simply arbitrary points in a poset.
- The stately model had a very clean mereological structure where all non-null states could be regarded as fusions of atomic states. While a Finean state space must include a null state, there needn't be atomic states from which all other states can be obtained by fusion.

- Because the stately model included only possible states, we couldn't always take fusions. By contrast, Fine allows for impossible states—indeed, a state space can have many impossible states—and fusions always exist by the completeness condition.
- Keep in mind that the modal structure in a modalized state space is treated as primitive. We do not determine whether a state in S is possible or impossible by 'looking within' the state itself.

2 Truthmaker semantics

We'll continue working with the simple non-modal language from last time:

Definition 10. The **basic language** $\mathcal{L}$ has the following syntax:

$p ::= \mathsf{A}, \mathsf{B}, \mathsf{C}, ...$
$\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi)$

Let $At_{\mathcal{L}} = \{\mathsf{A}, \mathsf{B}, ...\}$ be the (infinite) set of atoms in $\mathcal{L}$, and $S_{\mathcal{L}}$ be the set of all well-formed sentences in $\mathcal{L}$.

As before, sentences in $S_{\mathcal{L}}$ are evaluated with respect to a model:

Definition 11. A **state model** $\mathcal{M} = \langle S, \leqslant, |\cdot| \rangle$ for $\mathcal{L}$ consists of a state space $\langle S, \leqslant \rangle$ together with a **valuation** $|\cdot| : At_{\mathcal{L}} \to 2^S \times 2^S$ mapping each sentence letter $p \in At_{\mathcal{L}}$ to a pair $\langle |p|^+, |p|^- \rangle$ of **verifiers** and **falsifiers**.

Some more differences between Finean models and our earlier stately model:

- Whereas before there was no need for a separate valuation function, states are now simply points in S and don't directly encode any information about the truth or falsity of sentences, so we need $|\cdot|$ to provide this.
- For Fine, an atomic sentence can have multiple (exact) verifiers and/or multiple (exact) falsifiers.
- As things stand, there are no constraints between the verifiers and falsifiers of an atomic sentence—in fact, one and the same state might be both a verifier and falsifier for a sentence. Once we upgrade to modalized state spaces, however, we can impose certain plausible conditions.

Definition 12. A **modalized state model** $\mathcal{M} = \langle S, S^\lozenge, \leqslant, |\cdot| \rangle$ consists of a modalized state space together with a valuation.

For each $p \in At_{\mathcal{L}}$, we might now require the following:

- **Exclusivity:** No $s \in |p|^+$ and $t \in |p|^-$ are compatible.
- **Exhaustivity:** Every $u \in S^\lozenge$ is compatible with some $s \in |p|^+$ or $t \in |p|^-$.

Note that Exclusivity entails that no atomic sentence can have both a verifier and falsifier in a world, while Exhaustivity entails that every world includes either a verifier or falsifier for every atomic sentence. Of course, we needn't impose one or the other of these conditions if we want to allow for truth value gluts or gaps.

Fine gives an exact verification/falsification semantics:

Definition 13. Verification and falsification by a state, $s \Vdash \varphi$ and $s \dashV \varphi$, is defined recursively as follows (I now omit the E subscript):

$s \Vdash p$	iff	$s \in \lvert p \rvert^+$
$s \dashV p$	iff	$s \in \lvert p \rvert^-$
$s \Vdash \neg\varphi$	iff	$s \dashV \varphi$
$s \dashV \neg\varphi$	iff	$s \Vdash \varphi$
$s \Vdash \varphi \wedge \psi$	iff	$\exists t \exists u (s = t \oplus u$ and $t \Vdash \varphi$ and $u \Vdash \psi)$
$s \dashV \varphi \wedge \psi$	iff	$s \dashV \varphi$ or $s \dashV \psi$
$s \Vdash \varphi \vee \psi$	iff	$s \Vdash \varphi$ or $s \Vdash \psi$
$s \dashV \varphi \vee \psi$	iff	$\exists t \exists u (s = t \oplus u$ and $t \dashV \varphi$ and $u \dashV \psi)$

Some features and consequences of this semantics noted by Fine:

- $\varphi \wedge \varphi$ needn't have the same verifiers as φ. Suppose $|\mathsf{A}|^+ = \{a\}$ and $|\mathsf{B}|^+ = \{b\}$. While the verifiers for $\mathsf{A} \vee \mathsf{B}$ are $\{a, b\}$, the verifiers for $(\mathsf{A} \vee \mathsf{B}) \wedge (\mathsf{A} \vee \mathsf{B})$ are $\{a, b, a \oplus b\}$, which differ so long as neither a nor b is part of the other.

  This odd consequence can be avoided by working with the "inclusive" version of truthmaker semantics obtained by replacing the falsification clause for conjunction and verification clause for disjunction with the following alternatives:

$s \dashV \varphi \wedge \psi$	iff	$s \dashV \varphi$ or $s \dashV \psi$ or $s \dashV \varphi \vee \psi$
$s \Vdash \varphi \vee \psi$	iff	$s \Vdash \varphi$ or $s \Vdash \psi$ or $s \Vdash \varphi \wedge \psi$

  On this inclusive semantics, the sets of verifiers and falsifiers for any sentence are closed under fusions so long as the sets of verifiers and falsifiers for every atomic sentence are.

- While exact verification/falsification isn't hereditary (as mentioned last time), it is hereditary for all sentences in the language when verification/falsification for sentence letters is hereditary.

- As mentioned last time, exact verifiers needn't be minimal. Fine makes an important observation about this:

> There has been a persistent tendency in the literature (we might call it 'minimalitis') to start off with a hereditary notion of verification and then attempt to get the corresponding notion of minimal verification, or some variant of it, to do the work of exact verification (as in the account of 'exemplification' in Kratzer 2014). But if I am correct, all such attempts are doomed to failure. The relevant sense in which an exact verifier is wholly relevant to the statement it makes true is not one which requires that no part of the verifier be redundant but is one in which each part of the verifier can be seen to play an active role in verifying the statement. (p. 564; consider $a \oplus b$ as a verifier for $\mathsf{A} \vee (\mathsf{A} \wedge \mathsf{B})$)

- Can define the notions of *inexact* and *loose* verification in terms of exact verification (Fine claims the reverse definitions are impossible):

Definition 14. s **inexactly verifies** φ, $s \Vert\!> \varphi$, iff for some state $s' \leqslant s$, $s' \Vdash \varphi$.

Definition 15. s **loosely verifies** φ, $s \models \varphi$, iff s is incompatible with each s' such that $s' \dashV \varphi$.

> Thus the different notions of verification—loose, inexact, and exact—involve a progressively greater commitment to what might be involved in the verification of a given statement; and we obtain the greatest flexibility in developing a theory of verification by taking the exact notion as primitive and seeing the other notions as off-shoots of the exact notion. (p. 565)

- φ and ψ can have the same verifiers even though $\neg\varphi$ and $\neg\psi$ do not. Suppose $|\mathsf{A}|^+ = \{a\}$, $|\mathsf{B}|^+ = \{b\}$, $|\mathsf{C}|^+ = \{c\}$, $|\mathsf{A}|^- = \{a'\}$, $|\mathsf{B}|^- = \{b'\}$, and $|\mathsf{C}|^- = \{c'\}$. The verifiers for both $\mathsf{A} \wedge (\mathsf{B} \vee \mathsf{C})$ and $(\mathsf{A} \wedge \mathsf{B}) \vee (\mathsf{A} \wedge \mathsf{C})$ are $\{a \oplus b, a \oplus c\}$. However, while the verifiers for $\neg(\mathsf{A} \wedge (\mathsf{B} \vee \mathsf{C}))$ are $\{a', b' \oplus c'\}$, the verifiers for $\neg((\mathsf{A} \wedge \mathsf{B}) \vee (\mathsf{A} \wedge \mathsf{C}))$ are $\{a', a' \oplus b', a' \oplus c', b' \oplus c'\}$.

- So we need the double recursion. It also means that the notion of *proposition* is more complicated...

Definition 16. The **unilateral proposition** expressed by φ is the set of its verifiers $[\![\varphi]\!]_u = \{s \in S : s \Vdash \varphi\}$.

Definition 17. The **conjunction** $P \wedge Q$ of two unilateral propositions P and Q is $\{s \oplus t : s \in P, t \in Q\}$. The **disjunction** $P \vee Q$ of two unilateral propositions P and Q is $P \cup Q$.

Definition 18. The **bilateral proposition** expressed by φ is the pair $[\![\varphi]\!]_b = \langle \{s \in S : s \Vdash \varphi\}, \{s \in S : s \dashV \varphi\} \rangle$.

Definition 19. The **negation** $\neg(P, P')$ of a bilateral proposition (P, P') is (P', P). The **conjunction** $(P, P') \wedge (Q, Q')$ of two bilateral propositions (P, P') and (Q, Q') is $(P \wedge Q, P' \vee Q')$. The **disjunction** $(P, P') \vee (Q, Q')$ of two bilateral propositions (P, P') and (Q, Q') is $(P \vee Q, P' \wedge Q')$.

As mentioned last time, one of the most interesting features of truthmaker semantics from a logical point of view is that it naturally gives rise to multiple notions of consequence.

Definition 20. P is a **conjunctive part** of Q iff $Q = P \wedge R$ for some R.

Definition 21. P is a **disjunctive part** of Q iff $Q = P \vee R$ for some R.

We can define two different consequence relations in terms of these notions:

Definition 22. $P \models_C Q$ iff Q is a conjunctive part of P.

Definition 23. $P \models_D Q$ iff P is a disjunctive part of Q.

Within the classic possible worlds setting, these relations extensionally coincide. But within truthmaker semantics, they come apart. Working with the unilateral conception of propositions, the disjunctive part-based consequence relation $\models_D$ corresponds most closely to the classical notion, as $P \models_D Q$ iff every verifier for P is a verifier for Q (though while we get disjunctive weakening, we don't even get conjunction elimination). On the other hand, $P \models_C Q$ only if (and iff under simplifying assumptions) the following two conditions obtain:

- every verifier of P includes a verifier of Q
- every verifier of Q is included in a verifier of P

This latter relation of conjunctive part "corresponds to the intuitive notion of *partial content*—of what is conveyed, in whole or part, by what is said." (p. 566)

Some pretty interesting big-picture observations:

> The existence of the two relations of consequence may be of some methodological significance to the study of linguistics. For it is often assumed that intuitions of validity provide a key piece of data (some might think, *the* key piece of data) in the construction of a formal semantics for natural language. But, if I am right, then we should be somewhat more sensitive to the different inferential relationships that might be in play and it will be of particular importance to distinguish the subclass of inferential relationships that preserve content (as in the example above) and not merely truth. (p. 566)

> It will have been noted that in specifying the verifiers of truth-functionally complex statements, we have not restricted ourselves to possible states...The present point of view is that there is nothing in the general notion of content or meaning or in the most general logical devices that requires us to draw the distinction between possible and impossible states. This freedom from the modal thinking that has been so characteristic of the more usual approaches to semantics is, I believe, one of the most distinctive and liberating aspects of the present approach. (p. 566)

3 Quantification

Let us now extend the basic language $\mathcal{L}$ with quantifiers:

Definition 24. The **quantificational language** $\mathcal{L}^+$ has the following syntax:

$t ::= x \mid \mathsf{c}$
$\varphi ::= \mathsf{P}(t_1, ..., t_n) \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid \forall x\varphi \mid \exists x\varphi$

Let $At_{\mathcal{L}^+}$ be the set of atomic formulae in $\mathcal{L}^+$, and $S_{\mathcal{L}^+}$ be the set of all well-formed sentences in $\mathcal{L}^+$.

To provide a truthmaker semantics for this language, state models can be extended with a set of individuals and a valuation function for dealing with predications:

Definition 25. A **state model** $\mathcal{M} = \langle S, A, \leqslant, |\cdot|\rangle$ for $\mathcal{L}^+$ consists of a state space $\langle S, \leqslant\rangle$, a nonempty set of individuals A, and a valuation $|\cdot|$ mapping each n-place predicate P and n individuals $a_1, ..., a_n \in A$ to a pair $\langle|\mathsf{P}, a_1, ..., a_n|^+, |\mathsf{P}, a_1, ..., a_n|^-\rangle$ of verifiers and falsifiers.

Now suppose the language has constants $\mathsf{a}_1, \mathsf{a}_2, ...$ referring to each of the individuals $a_1, a_2, ... \in A$. First pass semantics (note that the clauses for the Boolean connectives are as before):

Definition 26. $s \Vdash \varphi$ and $s \dashv\!\!| \varphi$ is defined for closed atomic sentences and quantificational formulae as follows:

$$
\begin{array}{lll}
s \Vdash \mathsf{P}(\mathsf{a}_1, ..., \mathsf{a}_n) & \text{iff} & s \in |\mathsf{P}, a_1, ..., a_n|^+ \\
s \dashv\!\!| \mathsf{P}(\mathsf{a}_1, ..., \mathsf{a}_n) & \text{iff} & s \in |\mathsf{P}, a_1, ..., a_n|^- \\
s \Vdash \forall x\varphi(x) & \text{iff} & \exists s_1 \exists s_2 ...(s_1 \Vdash \varphi(\mathsf{a}_1), s_2 \Vdash \varphi(\mathsf{a}_2), ... \text{ and} \\
 & & s = s_1 \oplus s_2 \oplus ...) \\
s \dashv\!\!| \forall x\varphi(x) & \text{iff} & s \dashv\!\!| \varphi(\mathsf{a}_i) \text{ for some } a_i \\
s \Vdash \exists x\varphi(x) & \text{iff} & s \Vdash \varphi(\mathsf{a}_i) \text{ for some } a_i \\
s \dashv\!\!| \exists x\varphi(x) & \text{iff} & \exists s_1 \exists s_2 ...(s_1 \dashv\!\!| \varphi(\mathsf{a}_1), s_2 \dashv\!\!| \varphi(\mathsf{a}_2), ... \text{ and} \\
 & & s = s_1 \oplus s_2 \oplus ...)
\end{array}
$$

However, this assumes a fixed domain of existing individuals and we often want to allow for varying domains.

One solution is to introduce the *possibilist* "outer" quantifiers Π and Σ ranging over all of A and an existence predicate E, and then treating $\forall x\varphi(x)$ and $\exists x\varphi(x)$ as equivalent to $\Pi x(\mathsf{E}x \supset \varphi(x))$ and $\Sigma x(\mathsf{E}x \wedge \varphi(x))$.

A second solution is to assume that for each $B \subseteq A$, there is a *totality state* τ_B that obtains when B are exactly the individuals that there are. We can then adjust our quantifier clauses along the following lines:

$$
\begin{array}{lll}
s \Vdash \forall x\varphi(x) & \text{iff} & \exists B \subseteq A \text{ composed of individuals named by } \mathsf{b}_1, \mathsf{b}_2, ... \\
 & & \text{and } \exists s_1 \exists s_2 ...(s_1 \Vdash \varphi(\mathsf{b}_1), s_2 \Vdash \varphi(\mathsf{b}_2), ... \text{ and} \\
 & & s = \tau_B \oplus s_1 \oplus s_2 \oplus ...) \\
s \dashv\!\!| \forall x\varphi(x) & \text{iff} & \exists B \subseteq A \text{ containing an individual named by } \mathsf{b}_i \\
 & & \text{and } \exists s'(s' \dashv\!\!| \varphi(\mathsf{b}_i) \text{ and } s = \tau_B \oplus s')
\end{array}
$$

Alternatively, we might think of τ_B as a *precondition* for $s_1 \oplus s_2 \oplus ...$ to verify $\forall x\varphi(x)$ or for s' to falsify $\forall x\varphi(x)$. We could then take the verifiers and falisifiers for $\forall x\varphi(x)$ to be ordered pairs of the form $\langle\tau_B, s\rangle$, whose first component is a precondition and second component is a "post-condition" or "verifier/falsifier proper".

Truthmakers and Tautological Entailment

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1 Tautological entailment

Interestingly, part of the origin story of truthmaker semantics goes back to work on relevance logic in the 1960s. At least in these early days, relevance logicians were concerned with blocking "fallacies of relevance", especially the following:

Ex Falso Quodlibet (EFQ): $\varphi \wedge \neg\varphi \to \psi$.

Anderson & Belnap (1962) introduced the notion of *tautological entailment* in an influential attempt to capture a requirement of relevance alongside truth preservation.

Definition 1. Given a basic sentential language $\mathcal{L}$,

- A **literal** α is a sentence letter p or negated sentence letter $\neg p$.
- A **primitive disjunction** is a disjunction $\alpha_1 \vee ... \vee \alpha_n$ of literals.
- A **primitive conjunction** is a conjunction $\alpha_1 \wedge ... \wedge \alpha_n$ of literals.
- A **primitive entailment** $\varphi \to \psi$ relates a primitive conjunction φ to a primitive disjunction ψ.

Definition 2. A primitive entailment $\varphi \to \psi$ is **explicitly tautological** if some conjoined literal in φ is identical to some disjoined literal in ψ.

Examples: $\mathsf{A} \wedge \neg\mathsf{A} \to \mathsf{A}$ is explicitly tautological while $\mathsf{A} \wedge \neg\mathsf{A} \to \mathsf{B}$ is not. Likewise, $\mathsf{A} \to \mathsf{A} \vee \neg\mathsf{A}$ is explicitly tautological while $\mathsf{B} \to \mathsf{A} \vee \neg\mathsf{A}$ is not.

Anderson & Belnap's gloss:

> Such entailments may be thought of as satisfying the classical dogma that for A to entail B, B must be "contained" in A. (p. 12)

While $\mathsf{A} \to \mathsf{A} \vee \mathsf{B}$ is a primitive entailment, is $\mathsf{A} \vee \mathsf{B}$ really "contained" in A? Anderson & Belnap address this in footnote 2 of their paper, where they compare explicitly tautological entailments to Parry's (1933) related notion of *analytische Implikation* that invalidates disjunctive weakening:

> But there is surely a sense in which $\mathsf{A} \vee \mathsf{B}$ is "contained" in A; viz., the sense in which the concept Sibling (which is most naturally defined as Brother-or-Sister) is contained in the concept Brother. Certainly "All brothers are siblings" would have been regarded as analytic by Kant.

This isn't very convincing. I suppose that in this cherry-picked example we might say that *John is my sibling* (i.e., *John is my brother or sister*) is contained in *John is my brother*. But what about, say, *John is my brother or Sally is my sister*? In what sense is this sentence contained in *John is my brother*?

In any case, let us press on. The notion of being explicitly tautological applies only to primitive entailments so needs to be generalized.

Definition 3. An entailment $\varphi \to \psi$ is in **normal form** when φ is a disjunction $\varphi_1 \vee ... \vee \varphi_n$ of primitive conjunctions and ψ is a conjunction $\psi_1 \wedge ... \wedge \psi_m$ of primitive disjunctions.

Example: $(\mathsf{A} \wedge \mathsf{B}) \vee \neg\mathsf{A} \to (\mathsf{A} \vee \neg\mathsf{A}) \wedge (\mathsf{B} \vee \neg\mathsf{A})$ is in normal form.

Given the following replacement rules, every "first-degree entailment" (an entailment where φ and ψ are truth-functional (i.e., they do not themselves contain $\to$)) can be converted into normal form:

- *Commutativity:* $\varphi \wedge \psi$ and $\psi \wedge \varphi$ are intersubstitutable, as are $\varphi \vee \psi$ and $\psi \vee \varphi$.
- *Associativity:* $(\varphi \wedge \psi) \wedge \chi$ and $\varphi \wedge (\psi \wedge \chi)$ are intersubstitutable, as are $(\varphi \vee \psi) \vee \chi$ and $\varphi \vee (\psi \vee \chi)$.
- *Distributivity:* $\varphi \wedge (\psi \vee \chi)$ and $(\varphi \wedge \psi) \vee (\varphi \wedge \chi)$ are intersubstitutable, as are $\varphi \vee (\psi \wedge \chi)$ and $(\varphi \vee \psi) \wedge (\varphi \vee \chi)$.
- *Double negation:* φ and $\neg\neg\varphi$ are intersubstitutable.
- *De Morgan's laws:* $\neg(\varphi \wedge \psi)$ and $\neg\varphi \vee \neg\psi$ are intersubstitutable, as are $\neg(\varphi \vee \psi)$ and $\neg\varphi \wedge \neg\psi$.

Definition 4. A (first-degree) entailment $\varphi \to \psi$ is a **tautological entailment** if it can be converted into normal form $\varphi_1 \vee ... \vee \varphi_n \to \psi_1 \wedge ... \wedge \psi_m$ where each $\varphi_i \to \psi_j$ is explicitly tautological.

Example: $\neg(\mathsf{A} \vee \mathsf{B}) \wedge \neg\mathsf{C} \to \neg(\mathsf{A} \vee \mathsf{C})$ is a tautological entailment as it can be converted into $\neg\mathsf{A} \wedge \neg\mathsf{B} \wedge \neg\mathsf{C} \to \neg\mathsf{A} \wedge \neg\mathsf{C}$ and both $\neg\mathsf{A} \wedge \neg\mathsf{B} \wedge \neg\mathsf{C} \to \neg\mathsf{A}$ and $\neg\mathsf{A} \wedge \neg\mathsf{B} \wedge \neg\mathsf{C} \to \neg\mathsf{C}$ are explicitly tautological.

Example: $(\mathsf{A} \supset \mathsf{B}) \wedge (\mathsf{B} \supset \mathsf{C}) \rightarrow \mathsf{A} \supset \mathsf{C}$ isn't a tautological entailment (i.e., the material conditional isn't transitive).

Example: $(\mathsf{A} \supset \mathsf{B}) \wedge \mathsf{A} \rightarrow \mathsf{B}$ isn't a tautological entailment (i.e., *modus ponens* for $\supset$ can fail).

Example: $(\mathsf{A} \vee \mathsf{B}) \wedge \neg \mathsf{A} \rightarrow \mathsf{B}$ isn't a tautological entailment (i.e., disjunctive syllogism can fail).

The invalidity of disjunctive syllogism is particularly important for the relevantist project, as it blocks the following Lewis-style argument for EFQ:

1	$\varphi \wedge \neg\varphi$	Premise
2	φ	$\wedge$ Elim: 1
3	$\neg\varphi$	$\wedge$ Elim: 1
4	$\varphi \vee \psi$	$\vee$ Intro: 2
5	ψ	Disjunctive Syllogism: 4,3

I've always found it extremely odd that Anderson & Belnap pin the trouble on disjunctive syllogism rather than the use of disjunctive weakening at step 4, which introduces the alien subject matter contributed by ψ. If one is looking for fallacies of relevance, disjunctive weakening is the clear culprit. Interestingly, Anderson & Belnap address this in their paper and suggest that *or* has two possible meanings. Here is what they say:

> In rejecting the principle of disjunctive syllogism, we intend to restrict our rejection to the case in which the "or" is taken truth-functionally. In general and with respect to our ordinary reasonings this would not be the case; perhaps always when the principle is used in reasoning one has in mind an intensional meaning of "or", where there is relevance between the disjuncts. But for the intensional meaning of "or", it seems clear that the analogues of $\mathsf{A} \rightarrow \mathsf{A} \vee \mathsf{B}$ are invalid, since this would hold only if B was relevant to A; hence, there is a sense in which the real flaw in Lewis' argument is not a fallacy of relevance but rather a fallacy of ambiguity: [step 4] is valid only if the "$\vee$" is read truth-functionally, while [step 5] is valid only if the "$\vee$" is taken intensionally. (p. 19)

I don't really understand this. Anderson & Belnap are worried about fallacies of relevance and admit that disjunctive weakening is problematic when considerations of relevance are brought into play (when $\vee$ is read intensionally, whatever this means). So the focus on rejecting disjunctive syllogism is bizarre (and this consequence later came under scathing attack by Burgess 1983 and others).

2 Four-valued semantics

Anderson & Belnap (1975) and Belnap (1977) later developed an elegant four-valued logic for tautological entailment (Anderson & Belnap suggest a related matrix-theoretic semantics in their 1962 paper).

Let $\mathbf{4} = \{\emptyset, \{F\}, \{T\}, \{T, F\}\}$, where $\emptyset$ is neither true nor false, $\{F\}$ is false only, $\{T\}$ is true only, and $\{T, F\}$ is both true and false.

Definition 5. A **model** $\mathcal{M} = \langle V \rangle$ for $\mathcal{L}$ is a **valuation** $V : At_{\mathcal{L}} \rightarrow \mathbf{4}$ mapping each sentence letter $p \in At_{\mathcal{L}}$ to a truth value $V(p) \in \mathbf{4}$.

Definition 6. The **interpretation** $[\![\ ]\!]_{\mathcal{M}} : S_{\mathcal{L}} \rightarrow \mathbf{4}$ for the full language $\mathcal{L}$ is determined by the following truth tables:

¬	
{ }	{ }
{F}	{T}
{T}	{F}
{T,F}	{T,F}

∧	{ }	{F}	{T}	{T,F}
{ }	{ }	{F}	{ }	{F}
{F}	{F}	{F}	{F}	{F}
{T}	{ }	{F}	{T}	{T,F}
{T,F}	{F}	{F}	{T,F}	{T,F}

∨	{ }	{F}	{T}	{T,F}
{ }	{ }	{ }	{T}	{T}
{F}	{ }	{F}	{T}	{T,F}
{T}	{T}	{T}	{T}	{T}
{T,F}	{T}	{T,F}	{T}	{T,F}

Definition 7. A sentence $\varphi \in S_{\mathcal{L}}$ is **true** in $\mathcal{M}$ if $[\![\varphi]\!]_{\mathcal{M}} \in \{\{T\}, \{T, F\}\}$ and **false** in $\mathcal{M}$ if $[\![\varphi]\!]_{\mathcal{M}} \in \{\{F\}, \{T, F\}\}$.

The following three ways to define consequence over this semantics all turn out to be equivalent (Dunn 1976 provides an elegant proof of this involving switching the $\emptyset$ and $\{T, F\}$ values in an interpretation):

Definition 8. $\{\varphi_1, ..., \varphi_n\} \models_4 \psi$ iff any of the following conditions hold:

- *Truth preservation:* there is no model $\mathcal{M}$ such that $\varphi_1 \wedge ... \wedge \varphi_n$ is true in $\mathcal{M}$ but ψ is not.
- *Non-Falsity preservation:* there is no model $\mathcal{M}$ such that ψ is false in $\mathcal{M}$ but $\varphi_1 \wedge ... \wedge \varphi_n$ is not.

- *Truth + Non-Falsity preservation:* both of the above obtain.

Theorem 1. $\varphi_1 \wedge ... \wedge \varphi_n \to \psi$ is a tautological entailment iff $\{\varphi_1, ..., \varphi_n\} \models_4 \psi$.

3 Truthmaker semantics

This brings us to this week's reading in which van Fraassen (1969) offers an alternative semantic characterization of tautological entailment in terms of truthmakers. The paper begins with philosophical discussion of what kinds of facts we need to postulate to make sense of the idea that true/false sentences are *made true/false* by facts in the world. Executive summary of this opening part: (i) vF likes atomic facts but not negative facts; (ii) we need either conjunctive facts or disjunctive facts to account for the verification and falsification of conjunctions and disjunctions, and vF admits only conjunctive facts for convenience's sake; (iii) vF thinks 'fact' talk needn't involve ontological commitment:

> I propose that we retain our ontological neutrality, and treat facts as we do possibles: that is, explicate "fact" discourse in such a way that engaging in such discourse does not involve ontological commitment. This means that we must represent facts, relations among facts, and relations between facts and sentences; this representation can serve to explicate fact discourse without requiring the claim that it also represents a reality. (Indeed, such a claim would, if unqualified, be necessarily false; for we wish to explicate discourse about nonexistents and impossibles as well as existents.) The nature of the representation is of course dictated by methodological considerations; unlike the ontologist, we cannot be embarrassed by achieving parsimony at the cost of being arbitrary. (p. 481-2)

Moreover, (iv) facts aren't language-dependent—there needn't be an atomic sentence in the language corresponding to every atomic fact. vF offers two motivations for this treatment. First, if facts are language-independent, then we can avoid having to postulate special negative facts. As I discuss in more detail below, the fact that makes an atomic sentence Rab false is the triple $\langle \overline{R}, a, b \rangle$, where $\overline{R}$ is a relation that needn't be expressed by some predicate symbol in the language. Second, "facts have traditionally been held to be independent of what anyone may think, or say, or be able to say, about them" (p. 482).

Formally, vF works with a first-order predicate language $\mathcal{L}^+$, though he largely ignores the quantifiers and I'll bracket these off here. I also assume that the language has constants for each entity under consideration (and so I won't bring in variable assignments).

Definition 9. A (first-order) **model** $\mathcal{M} = \langle D, \mathcal{I} \rangle$ for $\mathcal{L}^+$ consists of a nonempty domain of entities D and an interpretation function $\mathcal{I}$ mapping each constant d in the language to an entity $d \in D$ and each n-ary predicate symbol R to a subset $\mathcal{I}(\mathsf{R}) \subseteq D^n$.

Example:

$$
\begin{aligned}
D &= \{\text{Alfonso}, \text{Billy}, \text{Clementine}\} \\
\mathcal{I}(\mathsf{a}) &= \text{Alfonso} \\
\mathcal{I}(\mathsf{b}) &= \text{Billy} \\
\mathcal{I}(\mathsf{c}) &= \text{Clementine} \\
\mathcal{I}(\mathsf{P}) &= \{\text{Alfonso}, \text{Clementine}\} \\
\mathcal{I}(\mathsf{R}) &= \{\langle \text{Alfonso}, \text{Billy} \rangle, \langle \text{Billy}, \text{Alfonso} \rangle\}
\end{aligned}
$$

Definition 10. A **complex** in $\mathcal{M}$ is any $(n+1)$-tuple $\langle R, d_1, ..., d_n \rangle$ whose first member R is an n-ary relation on D (this needn't be the interpretation $\mathcal{I}(\mathsf{R})$ of some predicate R of $\mathcal{L}^+$) and whose remaining components $d_1, ..., d_n$ are individuals in D.

Examples:
$\langle \{\text{Alfonso}, \text{Billy}\}, \text{Alfonso} \rangle$
$\langle \{\langle \text{Alfonso}, \text{Billy} \rangle, \langle \text{Billy}, \text{Alfonso} \rangle\}, \text{Alfonso}, \text{Clementine} \rangle$

Note that complexes, for vF, are *structured* in much the same way as propositions on structured meaning accounts.

Definition 11. $\langle R, d_1, ..., d_n \rangle$ **is the case in** $\mathcal{M}$ iff $\langle d_1, ..., d_n \rangle \in R$.

Example: While the complex $\langle \{\text{Alfonso}, \text{Clementine}\}, \text{Alfonso} \rangle$ is the case, $\langle \{\langle \text{Alfonso}, \text{Billy} \rangle, \langle \text{Billy}, \text{Alfonso} \rangle\}, \text{Alfonso}, \text{Clementine} \rangle$ is not.

Definition 12. A **fact** in $\mathcal{M}$ is a nonempty set e of complexes.

Definition 13. Fact e **is the case in** $\mathcal{M}$ iff each complex in e is the case.

Note that facts needn't be the case, which I suppose isn't a problem given that vF has explicitly disavowed realist commitments (and so his "facts" needn't be facts in the ordinary pre-theoretic sense).

Atomic facts are singleton sets including only one complex. The union of facts $e_1, ..., e_n$ is the *conjunctive fact* $e_1 \oplus ... \oplus e_n$. Strictly speaking, there are no negative facts (which would be the case when their constituent complexes fail to be the case) or disjunctive facts (which would be the case when one or more of their constituent complexes are the case). But vF could have set things up differently.

Definition 14. The truthmaker semantics for $\mathcal{L}^+$ assigns each sentence $\varphi \in S_{\mathcal{L}^+}$ a set of **truthmakers** $T(\varphi)$ and **falsemakers** $F(\varphi)$ in $\mathcal{M}$ as follows:

$$\begin{array}{lcl}
T(\mathsf{Rd}_1 ... \mathsf{d}_n) & = & \{\{\langle \mathcal{I}(\mathsf{R}), \mathcal{I}(\mathsf{d}_1), ..., \mathcal{I}(\mathsf{d}_n)\rangle\}\} \\
F(\mathsf{Rd}_1 ... \mathsf{d}_n) & = & \{\{\langle \overline{\mathcal{I}(\mathsf{R})}, \mathcal{I}(\mathsf{d}_1), ..., \mathcal{I}(\mathsf{d}_n)\rangle\}\} \\
T(\neg\varphi) & = & F(\varphi) \\
F(\neg\varphi) & = & T(\varphi) \\
T(\varphi \wedge \psi) & = & \{e \oplus e' : e \in T(\varphi), e' \in T(\psi)\} \\
F(\varphi \wedge \psi) & = & F(\varphi) \cup F(\psi) \\
T(\varphi \vee \psi) & = & T(\varphi) \cup T(\psi) \\
F(\varphi \vee \psi) & = & \{e \oplus e' : e \in F(\varphi), e' \in F(\psi)\}
\end{array}$$

Definition 15. A sentence $\varphi \in S_{\mathcal{L}^+}$ is **true** in $\mathcal{M}$ iff some fact $e \in T(\varphi)$ is the case in $\mathcal{M}$, and **false** in $\mathcal{M}$ iff some fact $e \in F(\varphi)$ is the case in $\mathcal{M}$.

This is very close to Fine's (2017) exact truthmaker semantics framed in terms of bilateral propositions. But there are some important differences, especially at the atomic level:

- Each atomic sentence has only one verifier and one falsifier.

- Moreover, the verifier and falsifier for an atomic sentence are highly structured facts determined by the interpretation of the predicate and constant symbols in this sentence.

- In our running example, the sole verifier for Pb is the atomic fact $\{\langle\{\text{Alfonso}, \text{Clementine}\}, \text{Billy}\rangle\}$ while the sole falsifier is the atomic fact $\{\langle\{\text{Billy}\}, \text{Billy}\rangle\}$.

- For another example, suppose that our language has three predicates P, Q, and R expressing incompatible properties (this incompatibility could be encoded in $\mathcal{I}$ for vF). One might think that the falsification set $F(\mathsf{Pa})$ should include $\{\langle\mathcal{I}(\mathsf{Q}), \text{Alfonso}\rangle\}$ and $\{\langle\mathcal{I}(\mathsf{R}), \text{Alfonso}\rangle\}$. But for vF, $F(\mathsf{Pa})$ contains only the single fact $\{\langle\overline{\mathcal{I}(\mathsf{P})}, \text{Alfonso}\rangle\}$. Note that Fine's more general approach would effectively allow us to take any of these three facts to be falsifiers, but vF's account has less freedom.

While vF provides an exact semantics, he also, like Fine, derives notions of inexact verification and falsification in terms of this:

Definition 16. Fact e **forces** fact e' iff $e' \subseteq e$. (Think of $\subseteq$ here as encoding parthood.)

Definition 17. $T^*(\varphi) = \{e : e \text{ forces some } e' \in T(\varphi)\}$ is the set of **wide truthmakers** for φ (i.e., the set of facts that make φ true in "the wider sense") and $F^*(\varphi) = \{e : e \text{ forces some } e' \in F(\varphi)\}$ is the set of **wide falsemakers** for φ.

Clearly we also have:

- A sentence $\varphi \in S_{\mathcal{L}^+}$ is true in $\mathcal{M}$ iff some fact $e \in T^*(\varphi)$ is the case in $\mathcal{M}$, and false in $\mathcal{M}$ iff some fact $e \in F^*(\varphi)$ is the case in $\mathcal{M}$.

As for defining a consequence relation for this semantics, vF realizes that this is where the interesting action is:

> Facts will hardly be of interest if they serve only to redefine truth in a model; if facts are to have a use, they must serve to define interesting new semantic relations. And here the most promising avenue of the approach would seem to be the replacement of the notion of "being true" by that of "being made true". Specifically, let us consider the relation of *semantic entailment* which is defined in terms of "being true". We say that A semantically entails B ($\mathsf{A} \Vdash \mathsf{B}$) if, whenever A is true, so is B. (More precisely: if A is true in model $\mathcal{M}$, then B is true in $\mathcal{M}$.) To this corresponds then the tighter relationship: Whatever makes A true, also makes B true. (p. 484-5)

This tighter relationship might be spelled out exactly or inexactly. First exactly:

Definition 18. $\varphi \Vdash \psi$ iff $T(\varphi) \subseteq T(\psi)$ in every model $\mathcal{M}$.

As vF notes, we then have $\mathsf{A} \Vdash \mathsf{A} \vee \mathsf{B}$ but $\mathsf{A} \wedge \mathsf{B} \nVdash \mathsf{A}$.

Next inexactly:

Definition 19. $\varphi \Vdash \psi$ iff $T^*(\varphi) \subseteq T^*(\psi)$ in every model $\mathcal{M}$.

Main result of the paper:

Theorem 2. $\varphi \to \psi$ is a tautological entailment iff $\varphi \Vdash \psi$.

Before exploring this connection in further detail, some general remarks:

- Despite providing an exact truthmaker semantics, vF ultimately thinks that bringing in facts is useful because of the consequence relation defined in terms of inexact truthmaking. Exact verification and falsification isn't really playing an essential role as the semantics could have been stated directly in terms of T^* and F^*.
- The notion of complexes/facts being the case or not isn't used in defining consequence.
- vF was standing at the doorstep of the containment relation defined in terms of exact settlement—whatever makes φ true (in the narrow sense) forces a fact that makes ψ true (in the narrow sense), and whatever makes ψ true is forced by a fact that makes φ true—but he didn't quite get there. This was pretty unfortunate as the notion of containment was what relevantists like Anderson & Belnap were trying to capture.

vF's proof of Thm 2 uses the following lemma:

Lemma 1. $T^*(\varphi) \subseteq T^*(\psi)$ iff $T(\varphi) \subseteq T^*(\psi)$ (i.e., every $e \in T(\varphi)$ forces some $e' \in T(\psi)$).

Proof: For the left-to-right direction, suppose $T^*(\varphi) \subseteq T^*(\psi)$. Since $T(\varphi) \subseteq T^*(\varphi)$ (given that forcing is reflexive), $T(\varphi) \subseteq T^*(\psi)$. For the right-to-left direction, suppose $T(\varphi) \subseteq T^*(\psi)$ and consider $e \in T^*(\varphi)$. We have $e' \subseteq e$ for some $e' \in T(\varphi)$. Since $e' \in T^*(\psi)$, we also have $e'' \subseteq e'$ for some $e'' \in T(\psi)$. But then $e'' \subseteq e$ and so $e \in T^*(\psi)$.

Note that the right hand side condition $T(\varphi) \subseteq T^*(\psi)$ of Lem 1 is one half of containment, which also requires that every $e \in T(\psi)$ is forced by some $e' \in T(\varphi)$. Adding this second condition blocks disjunctive weakening from counting as valid.

Here is vF's proof of the left-to-right direction of Thm 2:

Lemma 2. If $\varphi \to \psi$ is a tautological entailment, then $\varphi \Vdash \psi$.

Proof: First consider the case where $\varphi \to \psi$ is a primitive entailment, so φ is a primitive conjunction $\alpha_1 \wedge ... \wedge \alpha_n$ and ψ is a primitive disjunction $\beta_1 \vee ... \vee \beta_m$. Consider $e \in T^*(\varphi)$. We have $e_1 \oplus ... \oplus e_n \subseteq e$ where $e_1 \in T(\alpha_1)$, ..., $e_n \in T(\alpha_n)$. Since $\varphi \to \psi$ is explicitly tautological, $\alpha_i = \beta_j$ for some i, j and so there is $e_j \subseteq e$ such that $e_j \in T(\psi)$, and so $e \in T^*(\psi)$.

Next consider the case where $\varphi \to \psi$ is in normal form, so φ is a disjunction $\varphi_1 \vee ... \vee \varphi_n$ of primitive conjunctions and ψ is a conjunction $\psi_1 \wedge ... \wedge \psi_m$ of primitive disjunctions, where each $\varphi_i \to \psi_j$ is explicitly tautological, and so $\varphi_i \Vdash \psi_j$. To establish $\varphi \Vdash \psi$, it suffices to show the following:

- If $\varphi_1 \Vdash \psi$ and $\varphi_2 \Vdash \psi$ then $\varphi_1 \vee \varphi_2 \Vdash \psi$.
- If $\varphi \Vdash \psi_1$ and $\varphi \Vdash \psi_2$ then $\varphi \Vdash \psi_1 \wedge \psi_2$.

For the first of these, suppose $e \in T^*(\varphi_1 \vee \varphi_2)$. We have $e' \subseteq e$ for some $e' \in T(\varphi_1 \vee \varphi_2)$. But then either $e' \in T(\varphi_1)$ or $e' \in T(\varphi_2)$. In the first case, $e \in T^*(\varphi_1)$ and so $e \in T^*(\psi)$ given that $\varphi_1 \Vdash \psi$. In the second case, $e \in T^*(\varphi_2)$ and so $e \in T^*(\psi)$ given that $\varphi_2 \Vdash \psi$. (The proof of the conjunctive principle is similar.)

To prove Lem 2 for the general case where $\varphi \to \psi$ needn't be in normal form, we must show that the replacement rules needed to convert any first-degree entailment into normal form don't make a difference truthmaker-wise. It's fairly easy to see that intersubstitutable sentences have the same truthmaker sets.

Moving on to the right-to-left direction:

Lemma 3. If $\varphi \Vdash \psi$, then $\varphi \to \psi$ is a tautological entailment.

Proof: Suppose $\varphi \to \psi$ isn't a tautological entailment. Since the replacement rules don't make a difference, we can assume that $\varphi \to \psi$ is in normal form, so φ is a disjunction $\varphi_1 \vee ... \vee \varphi_n$ of primitive conjunctions and ψ is a conjunction $\psi_1 \wedge ... \wedge \psi_m$ of primitive disjunctions, where some $\varphi_i \to \psi_j$ isn't explicitly tautological. Let φ_i be the primitive conjunction $\alpha_1 \wedge ... \wedge \alpha_n$ and ψ_j be the primitive disjunction $\beta_1 \vee ... \vee \beta_m$, where these have no overlap. Clearly $\alpha_1 \wedge ... \wedge \alpha_n \nVdash \beta_1 \vee ... \vee \beta_m$, so $\varphi \nVdash \psi$.

4 Four-valued truthmaker semantics

Taking stock, Belnap established that tautological entailment coincides with truth preservation in a 4-valued logic:

- $\varphi \to \psi$ is a tautological entailment iff $\{\varphi\} \models_4 \psi$.

van Fraassen established that tautological entailment coincides with inexact consequence in a truthmaker semantics:

- $\varphi \to \psi$ is a tautological entailment iff $T^*(\varphi) \subseteq T^*(\psi)$ in every $\mathcal{M}$.

The transitivity of equivalence gives us the following connection between the two semantic characterizations:

- $\{\varphi\} \models_4 \psi$ iff $T^*(\varphi) \subseteq T^*(\psi)$ in every $\mathcal{M}$.

Can we make deeper sense of this connection between truthmakers and many-valued logic? Fine (2016) shows us how in §10 of "Angellic Content".

Basic idea (framed using vF's terminology, with relativization to a model left implicit):

Definition 20. Given any fact e, the ***e*-relativized 4-valued valuation** $V_e : At_{\mathcal{L}} \to \mathbf{4}$ is defined as follows:

$$\begin{array}{lll} V_e(p) = \{T, F\} & \text{iff} & e \in T^*(p) \text{ and } e \in F^*(p) \\ V_e(p) = \{T\} & \text{iff} & e \in T^*(p) \text{ and } e \notin F^*(p) \\ V_e(p) = \{F\} & \text{iff} & e \notin T^*(p) \text{ and } e \in F^*(p) \\ V_e(p) = \emptyset & \text{iff} & e \notin T^*(p) \text{ and } e \notin F^*(p) \end{array}$$

Definition 21. The ***e*-relativized interpretation** $[\![\ ]\!]_e : S_{\mathcal{L}} \to \mathbf{4}$ is obtained from V_e using Belnap's 4-valued truth tables.

A simple induction establishes the following for all $\varphi \in S_{\mathcal{L}}$:

$$\begin{array}{lll} [\![\varphi]\!]_e = \{T, F\} & \text{iff} & e \in T^*(\varphi) \text{ and } e \in F^*(\varphi) \\ [\![\varphi]\!]_e = \{T\} & \text{iff} & e \in T^*(\varphi) \text{ and } e \notin F^*(\varphi) \\ [\![\varphi]\!]_e = \{F\} & \text{iff} & e \notin T^*(\varphi) \text{ and } e \in F^*(\varphi) \\ [\![\varphi]\!]_e = \emptyset & \text{iff} & e \notin T^*(\varphi) \text{ and } e \notin F^*(\varphi) \end{array}$$

We can use this relativization method to directly prove the correspondence:

Theorem 3. $\{\varphi\} \models_4 \psi$ iff $T^*(\varphi) \subseteq T^*(\psi)$ in every $\mathcal{M}$.

Proof: For the left-to-right direction, suppose $T^*(\varphi) \not\subseteq T^*(\psi)$ in some $\mathcal{M}$. Then there is some fact e in $\mathcal{M}$ such that $e \in T^*(\varphi)$ but $e \notin T^*(\psi)$. Note that $[\![\varphi]\!]_e \in \{\{T\}, \{T, F\}\}$ but $[\![\psi]\!]_e \in \{\emptyset, \{F\}\}$, so $\{\varphi\} \not\models_4 \psi$.

For the right-to-left direction, suppose $\{\varphi\} \not\models_4 \psi$ so there is some 4-valued interpretation such that $[\![\varphi]\!] \in \{\{T\}, \{T, F\}\}$ and $[\![\psi]\!] \in \{\emptyset, \{F\}\}$. Let $e = \{\langle R, d_1, ..., d_n\rangle : [\![\mathsf{Rd}_1...\mathsf{d}_n]\!] \in \{\{T\}, \{T, F\}\}\} \cup \{\langle \overline{R}, d_1, ..., d_n\rangle : [\![\mathsf{Rd}_1...\mathsf{d}_n]\!] \in \{\{F\}, \{T, F\}\}\}$. Then $[\![\varphi]\!]_e = [\![\varphi]\!]$ for each $\varphi \in S_{\mathcal{L}}$, and so $e \in T^*(\varphi)$ but $e \notin T^*(\psi)$, i.e., $T^*(\varphi) \not\subseteq T^*(\psi)$.

Santorio's TMS for Conditionals

AS.150.645: Truthmakers
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1 A threefold tension

Three individually plausible constraints for counterfactuals:

Failure of Antecedent Strengthening: $\varphi \mathrel{\Box\!\!\rightarrow} \psi \not\models \varphi \wedge \chi \mathrel{\Box\!\!\rightarrow} \psi$.

Empirical support from felicitous Sobel sequences:

(1) If the US threw its weapons into the sea, there would be war. If the US and all other nuclear powers threw their weapons into the sea, there would not be war. Etcetera. (Lewis 1973)

Validity of Simplification: $\varphi \vee \psi \mathrel{\Box\!\!\rightarrow} \chi \models \varphi \mathrel{\Box\!\!\rightarrow} \chi, \psi \mathrel{\Box\!\!\rightarrow} \chi$.

Empirical support from examples such as the following:

(2) If Alice or Bob had come to the party, the party would be fun.
$\rightsquigarrow$ If Alice had come to the party, the party would be fun.
$\rightsquigarrow$ If Bob had come to the party, the party would be fun.

Further support from infelicitous sequences such as the following:

(3) #If Alice or Bob had come to the party, the party would be fun. If Bob had come, the party would be dreary.

Substitution of Logical Equivalents: $\varphi \mathrel{\Box\!\!\rightarrow} \psi \models \varphi' \mathrel{\Box\!\!\rightarrow} \psi$ when φ and φ' are logically equivalent.

Theoretical support from elegance and success of intensional possible worlds semantics as a general theory for modality.

However, if we maintain a Boolean semantics for disjunction, we cannot accept all three:

(4)
$\varphi \mathrel{\Box\!\!\rightarrow} \psi$
$\varphi \vee (\varphi \wedge \chi) \mathrel{\Box\!\!\rightarrow} \psi$ Substitution
$\varphi \wedge \chi \mathrel{\Box\!\!\rightarrow} \psi$ Simplification

Aside: While issues concerning Simplification have mostly been discussed in the literature on counterfactuals, they often generalize to indicatives as well. Paolo assumes a structurally uniform semantics for conditionals, though he focuses mostly on counterfactuals in his paper.

2 The standard theory

Theories of counterfactuals in the Lewisian-Stalnakerian tradition typically jettison Simplification while maintaining the other two principles. The classic theory based on comparative similarity $\leq$:

(5) **Selection function:**
$f_{\leq_w}(p)(w')$ iff $p(w') \wedge \forall w''(p(w'') \rightarrow w' \leq_w w'')$

(6) **Classic entry for *would*-counterfactuals:**
$[\![\varphi \mathrel{\Box\!\!\rightarrow} \psi]\!]^{\leq}(w)$ iff $\forall w'(f_{\leq_w}([\![\varphi]\!]^{\leq})(w') \rightarrow [\![\psi]\!]^{\leq}(w'))$

3 Rescuing simplification

Broadly speaking, semanticists interested in counterfactuals have responded to the Simplification data in one of two ways: they have either assimilated simplification inferences to scalar implicatures such as (7), or adopted a non-classical semantics for disjunction.

(7) Sarah talked to some of her students.
$\approx$ Sarah talked to some but not all of her students.

3.1 The pragmatic route

Basic idea behind most if not all implicature accounts (van Rooij 2010; Franke 2011; etc.): cooperative speakers will utter a counterfactual of the form $\varphi \vee \psi \mathrel{\Box\!\!\rightarrow} \chi$ only if the closest worlds for the purposes of evaluating it include both φ-worlds and ψ-worlds. Given this "Diversity Condition", as Paolo calls it, simplification results as a form of pragmatic inference.

Some preliminary support for implicature accounts from optionality of simplification inferences (predicting this 'optionality' is a desideratum of any account):

(8) Sarah talked to some of her students. In fact, she talked to all of them.

(9) If Spain had fought with the Axis or the Allies, she would have fought with the Axis. (McKay & van Inwagen 1977)

However, Paolo gives three arguments against implicature accounts:

- Unlike ordinary scalar implicatures, simplification inferences don't disappear in downward entailing environments.

(10) a. Sarah talked to some of her students.
≈ Sarah talked to some but not all of her students.
b. It's not the case that Sarah talked to some of her students.
≉ It's not the case that Sarah talked to some but not all of her students.
c. I doubt that Sarah talked to some of her students.
≉ I doubt that Sarah talked to some but not all of her students.
d. No teacher talked to some of her students.
≉ No teacher talked to some but not all of her students.

(11) a. Jane talked to Mary or Sue.
≈ Jane talked to Mary or Sue but not both. (exclusivity implicature)
b. It's not the case that Jane talked to Mary or Sue.
≉ It's not the case that Jane talked to Mary or Sue but not both.

(12) a. Mary may go to Paris or Berlin.
≈ Mary may go to Paris and she may go to Berlin. (free choice effect)
b. It's not the case that Mary may go to Paris or Berlin.
≉ It's not the case that: Mary may go to Paris and she may go to Berlin.

(Aside: less clear that free choice effects are cases of implicature.)

(13) a. It's not the case that, if Alice or Bob had come, the party would be fun.
⇝ It's not the case that, if Alice had come, the party would be fun.
⇝ It's not the case that, if Bob had come, the party would be fun.
b. I doubt that, if Alice or Bob had come, the party would be fun.
⇝ I doubt that, if Alice had come, the party would be fun.
⇝ I doubt that, if Bob had come, the party would be fun.
c. None of my friends would have had fun at the party if Alice or Bob had come.
⇝ None of my friends would have had fun at the party if Alice had come
⇝ None of my friends would have had fun at the party if Bob had come

I find this objection a bit confusing. First, we have ≉ but still ⇝ in implicature cases:

(14) It's not the case that Sarah talked to some of her students.
⇝ It's not the case that Sarah talked to some but not all of her students.

Second, Paolo claims that simplification inferences do disappear in at least the following sense:

(15) It's not the case that, if Alice or Bob had come, the party would be fun.
≉ It's not the case that: if Alice had come, the party would be fun, and that, if Bob had come, the party would be fun.

Third, I'm not sure about the judgments in (13) and (15) (especially if we add focal stress on *or* in lead sentence).

In any case, I find the remaining two objections more persuasive.

- Implicature accounts cannot accommodate simplification inferences for *probably*-conditionals.

(16) **Raffle.** Sarah bought 40 tickets in a 100-ticket raffle. The tickets she bought were numbered 31 to 70. The winning ticket was just picked. We are not told which ticket won, but we hear two rumors. On the first, the winning ticket is among tickets 1 to 70; on the second, it is among tickets 31 to 100.

If the winning ticket is between 1 and 70 or between 31 and 100, probably Sarah won.

This conditional is true under a simplification reading (interestingly, Paolo suggests it might have another reading on which it is false):

(17) If the winning ticket is between 1 and 70 or between 31 and 100, probably Sarah won.

$\rightsquigarrow$ If the winning ticket is between 1 and 70, probably Sarah won.
$\rightsquigarrow$ If the winning ticket is between 31 and 100, probably Sarah won.

But it's not clear how the implicature account can predict these inferences given the Diversity Condition. Can have >50% probability of Sarah winning within a set including both 1-70-worlds and 31-100 worlds without >50% probability of her winning within a set including only 1-70 worlds.

- The third objection arises from Lewis's (1973) famous argument for the intransitivity of counterfactuals:

(18) **Rivals.** Otto is Waldo's successful rival for Anna's affections. Waldo still tags around after Anna, but never runs the risk of meeting Otto. Otto was locked up at the time of the party, so that his going to it is a far-fetched supposition; but Anna almost did go.

The following counterfactuals hold:

If Anna had gone to the party, Waldo would have gone.

If Otto had gone to the party, Anna would have gone.

But this counterfactual fails to hold:

If Otto had gone to the party, Waldo would have gone.

This trio of judgments requires that all Anna-worlds are strictly closer than all Otto-worlds. (Paolo proves a more general result.)

But as Paolo notes, Simplification persists in this scenario:

(19) #If Otto or Anna had come to the party, the party would have been fun. If Otto had come, the party would have been dreary.

To predict this on the implicature account, the Diversity Condition would require that the closest Otto-or-Anna worlds include both Otto-worlds and Anna-worlds, which isn't possible.

3.2 The semantic route

An altogether different approach is to validate Simplification by offering an alternative-based semantic treatment of disjunction. Paolo focuses on Alonso-Ovalle's (2006) Hamblin-style semantics.

The theory in a nutshell:

(20) **Alternative semantics for counterfactuals with disjunctive antecedents:**
$[\![\varphi \vee \psi \;\Box\!\!\rightarrow \chi]\!]^{\leq}(w)$ iff for all $p \in \{[\![\varphi]\!]^{\leq}, [\![\psi]\!]^{\leq}\}$,
$\forall w'(f_{\leq_w}(p)(w') \rightarrow [\![\chi]\!]^{\leq}(w'))$

Paolo's only worry with Alonso-Ovalle's system is a compositional problem involving quantificational sentences.

(21) If every student read *Anna Karenina* or *War and Peace*, the world would be a better place.
$\rightsquigarrow$ If every student read *AK*, the world would be a better place.
$\rightsquigarrow$ If every student read *W&P*, the world would be a better place.
$\rightsquigarrow$ If some students read *AK* and the rest read *W&P*, the world would be a better place.

The system generates too few alternatives for the antecedent:

(22) Every student read *Anna Karenina* or *War and Peace*.

Lexical entries:

(23) $[\![\textit{Anna Karenina}]\!] = \{AK\}$, $[\![\textit{War and Peace}]\!] = \{W\&P\}$

(24) $[\![\textit{read}]\!] = \{\lambda y \lambda x \lambda w.x \text{ reads } y \text{ in } w\}$

(25) $[\![\textit{student}]\!] = \{\lambda x \lambda w.x \text{ is a student in } w\}$

(26) $[\![\textit{every}]\!] = \{\lambda P_{\langle e,\langle s,t\rangle\rangle} \lambda Q_{\langle e,\langle s,t\rangle\rangle} \lambda w.\text{all } P\text{-things are } Q\text{-things in } w\}$

The "*Or* Rule", simple version:

(27) $[\![\text{XP } \textit{or} \text{ YP}]\!] = [\![\text{XP}]\!] \cup [\![\text{YP}]\!]$

(28) $[\![\textit{Anna Karenina or War and Peace}]\!] = \{AK, W\&P\}$

Applying the "Hamblin Rule" (Pointwise Functional Application):

(29) $[\![\textit{read Anna Karenina or War and Peace}]\!] =$
$\{\lambda x \lambda w.x \text{ reads } AK \text{ in } w, \lambda x \lambda w.x \text{ reads } W\&P \text{ in } w\}$

(30) $[\![$*Every student read Anna Karenina or War and Peace*$]\!] =$
$\{\lambda w.$every student read AK in $w, \lambda w.$every student read $W\&P$ in $w\}$

We're missing the alternative where some students read AK and the rest read *W&P*, which is needed to derive the third inference in (21).

> It may be that we can fix the problem while remaining in a "local" framework. But the fix is not going to be trivial [footnote: The reason: to generate the right propositional alternatives, we need information about lexical items that take scope above disjunction when we are computing the meaning of the disjunctive phrase. For example, we would need the semantics to somehow "see" that there is a universal quantifier above when computing the meaning of the complex predicate $\lambda_1.[x_1$ read W&P or x_1 read AK]. It is unclear how this can be done compositionally.]. Rather than pursuing this, in the next section I pursue a "global" account, where propositional alternatives are generated at the end of the compositional computation. (p. 533)

4 The positive account

Paolo's own truthmaker account, rough version:

(31) **Truthmaker semantics for counterfactuals:**
$[\![\varphi \mathrel{\Box\!\!\rightarrow} \psi]\!]^{\leq}(w)$ iff for every way p for φ to be true,
$\forall w'(f_{\leq_w}(p)(w') \rightarrow [\![\psi]\!]^{\leq}(w'))$

Truthmakers for Paolo are defined on the basis of Katzir's (2007) theory of syntactic alternatives (as opposed to, say, Horn scales). The algorithm:

- Rough idea from Katzir: the set of alternatives ALT_φ to a sentence φ are all and only those sentences that are relevant in a context and no more complex than φ (i.e., we can derive them from φ via deletion or replacement with syntactic items from a given substitution source).

- Given the set of alternatives ALT_φ for the antecedent φ, we first determine the stable subsets which are consistent with the negations of their non-members: "The intuition is that a set of alternatives is stable just in case it contains enough information to stand alone—even if all other alternatives are false, it is still consistent to suppose that all sentences in the set are true" (p. 535):

  (32) $\Gamma \subseteq ALT_\varphi$ is *stable* iff $\Gamma \cup \{\neg\varphi : \varphi \in ALT_\varphi \setminus \Gamma\}$ is consistent.

- Next, we restrict attention to the minimal stable sets:

  (33) $\Gamma \subseteq ALT_\varphi$ is *minimally stable* iff Γ is stable and there is no Γ' such that Γ' is stable and $\Gamma' \subsetneq \Gamma$.

- Finally, we define the truthmakers for φ as the propositions expressed by the conjunctions of minimally stable sets that are stronger than $[\![\varphi]\!]$:

  (34) p is a *truthmaker* for φ iff $p = [\![\bigwedge\Gamma]\!]$ for some minimally stable set $\Gamma \subseteq ALT_\varphi$ and p entails $[\![\varphi]\!]$.

  (Note that this assumes we can determine the content $[\![\varphi]\!]$ prior to determining the truthmakers for φ; pace Fine.)

Putting this to work:

(35) Otto or Anna went to the party.

In this case, $ALT_{O\vee A} = \{O \vee A, O \wedge A, O, A\}$. The stable subsets are $\{O \vee A, O \wedge A, O, A\}$, $\{O \vee A, O\}$, and $\{O \vee A, A\}$, of which the latter two are minimal. The potential truthmakers for $O \vee A$ are then $[\![(O \vee A) \wedge O]\!]$ and $[\![(O \vee A) \wedge A]\!]$, i.e., $[\![O]\!]$ and $[\![A]\!]$. These are in fact genuine truthmakers as both entail $[\![O \vee A]\!]$.

Why restrict attention to only minimal stable sets? Because of cases like the following:

(36) **Plane Crash.** The three passengers in a small plane, contrary to the pilot's recommendations, clustered on the left-hand side of the plane because they enjoyed sitting together. As a result, the plane was unbalanced and crashed.

If some passengers had sat on the right-hand side, the plane would not have crashed.

Why the entailment condition? It screens off irrelevant alternatives. See Paolo's case in footnote 37 where John is added to the party crew.

As Paolo shows, his algorithm delivers a better result for the book case (p. 538-9).

Yablo's If-Thenism

AS.150.645: Truthmakers
Johns Hopkins University, Fall 2020

1 Hostage crises

> Philosophy is often held to be in crisis. I don't know about that; but certainly it is beset with cris*es*. I call them *hostage* crises because they involve a (relatively) thin, innocent claim and a (relatively) weighty, debatable one; the first is hostage to the second in that the second must hold or the first fails. (p. 115)

Examples (schematic notation: $\varphi \Rightarrow \psi$):

(1) I am thinking. ⇒ There is a thinking substance.

(2) This is a hand. ⇒ There are material objects.

(3) Usian Bolt won gold. ⇒ Technology will not be developed within the next six years that turns up banned substances in UB's urine sample.

(4) I'm thinking about Thales. ⇒ There was such a person as Thales.

(5) The number of Jupiter's moons is 79. ⇒ There are numbers.

(6) [The Peano axioms] ⇒ PA is consistent.

Possible responses:

- *Defiance:* the stronger claim φ is in fact likelier than the weaker claim ψ.
- *Skepticism:* the stronger claim φ is not as likely as we thought. (cf. Field on numbers)
- *Boosterism:* the weaker claim ψ is likelier than we thought. (cf. Moore against external world sceptics)

Yablo pursues the skeptical route, using the "understandable confusion" strategy: our assessment of φ is really aimed at a weaker claim φ^* in the vicinity that doesn't entail ψ, where the obviousness of φ^* is wrongly projected onto φ.

If/Thenism: $\varphi^* =$ *If* ψ *then* φ.

2 Hogan's (1984) if/thenism

Sets exist > [Scientifc Law], where > is the counterfactual connective.

Problems:

- Interference problem: what's to say that if sets were to exist the laws would be unchanged? Perhaps the miracle needed to bring sets into existence would bring all sorts of other changes in its wake. Hogan argues that the non-causal nature of abstract objects like sets precludes interference. However, Baker (2003) replies that causal independence is one thing, counterfactual independence another.
- Embedding worries: counterfactual if/thenism seems inappropriate for cases where φ occurs in certain embedded environments:

(7) Abraham hopes that the number of righteous men in Sodom and Gomorrah is ≥ 10.

> The city will not be spared after all, it turns out, if the ten men would *still* be righteous in the presence of numbers—ours is a jealous God who wants men be driven criminally insane by the existence of immaterial objects beyond himself. Here too we have a claim the practice treats as obvious (*Abraham hopes the number of righteous men is at least 10*) held hostage to an issue (*how God feels about counterfactual righteousness*) that the practice takes no notice of. (p. 122)

(8) Sodom and Gomorrah were destroyed because their righteous inhabitants numbered less than 10.

> If we don't want to attribute the cities' destruction to the size of an *actual* number—the number of righteous men—we shouldn't want to attribute it either to the size of a *would-be* number: the size a number of that description *would* have had, had there been numbers. Sodom and Gomorrah were destroyed because of an actual fact—the shortage of righteous men—not a conditional one about would-be numbers. It is not just that more men might have been righteous, had there been numbers. What is doing the causal work is the fact *underlying* the counterfactual, namely, that there weren't enough righteous men. (p. 123)

3 A better option: remainders

Remainder: $\varphi \sim \psi$ (read: *φ, except maybe not ψ*; *φ, ignoring the bit about ψ*; *φ, but possibly for ψ*; cf. Wittgenstein on intentional action; Goodman's "surplus content")

(9) Every Justice spoke up, with the possible exception of Thomas.
= Every Justice spoke up $\sim$ Thomas spoke up
= Every other Justice spoke up.

(10) Kennedy was killed by someone other than Oswald, or indeed by Oswald.
= Kennedy was killed by someone other than Oswald $\sim$ Oswald didn't kill Kennedy
= Kennedy was killed.

(11) Pete won, ignoring the possibility that he folded.
= Pete won $\sim$ Pete called
= Pete had the better hand.

> I like to think of $\sim$ as undoing the effect of conjunction, or better, as the operator such that conjunction (assuming φ implies ψ) undoes *its* effect: $(\varphi \sim \psi)\&\psi$ is true in the same worlds as φ. (footnote 17, p. 125)

How to determine remainders? Brief sketch of an outline of a truth-conditional theory:

- *Agreement:* $\varphi \sim \psi$ is true/false in a "home" ψ-world w iff φ is true/false in w.
- *Reasons:* $\varphi \sim \psi$ is true/false in a ψ-world w for the same reason(s) that φ is true/false in w given ψ.

  (This is the thorny bit. Yablo tries to clarify *Reasons* in a footnote in terms of targeted truthmakers but it's unintelligible (to me at least) without reading his other work. We could look into this more if there's interest in the seminar.)
- *Integrity:* $\varphi \sim \psi$ is true/false in an "away" $\neg\psi$-world w for the same reason(s) as it was true/false at home.
- *Projection:* $\varphi \sim \psi$ is true/false in a $\neg\psi$-world w iff it has reason(s) to be true/false there.

Incremental If/Thenism: $\varphi^* =$ *If ψ then φ* $= \varphi \sim \psi$.

- This avoids the worries with Hogan-style counterfactual if/thenism.
- Yablo notes that interpreting conditionals in terms of remainders is at odds with existing theories of conditionals. He makes it seem like we have the following choice: (partially) abandon existing accounts of conditionals, or give up on if/thenism. However, we could take φ^* to be $\varphi \sim \psi$ without also taking this to be equivalent to *If ψ then φ*. In this way, we could resist an incremental account of conditionals while still implementing the understandable confusion strategy in terms of remainders (though the term "if/thenism" would become a misnomer).

4 Conditionals and remainders

But why resist an incremental theory of conditionals? Yablo offers some considerations for thinking that conditionals can at least sometimes be read incrementally:

- Remainders and conditionals turn up in a lot of the same places in the philosophy and linguistics literature. For example, Hogan engages in remainder talk in presenting his counterfactual if/thenism.
- Remainders are apt to be formulated in conditional terms:

  (12) If Thomas spoke up, then all the Justices spoke up.

  (13) If Oswald didn't shoot Kennedy, then someone else did.

  (14) If Pete called, he won.
- Conditional test for non-catastrophic presupposition failure depends on incremental reading.

  (15) a. The king of France is sitting in this chair.
  b. Even if France has a king, still, he is not sitting in this chair.
- Conditionals and remainders respect *modus ponens* and centering:

$$\frac{\textit{If } \varphi \textit{ then } \psi \qquad \varphi}{\psi} \qquad \frac{\psi \sim \varphi \qquad \varphi}{\psi}$$

$$\frac{\varphi\&\psi}{\textit{If } \varphi \textit{ then } \psi} \qquad \frac{\varphi\&\psi}{\psi \sim \varphi}$$

- The strongest consideration in favor of incremental conditionality comes from pairs of conditionals *If* φ *then* ψ and *If* φ *then* χ, where ψ and χ are equivalent in the φ-region of logical space. The truth values of these conditionals can come apart (*pace* standard theories), suggesting that the incremental reading is the only one available. Here are a few of Yablo's examples:

(16) a. If that guy is Smith's murderer, then Smith's murder is insane.
$\rightsquigarrow$ That guy is insane.
b. If that guy is Smith's murderer, then that guy is insane.
$\rightsquigarrow$ Smith's murderer is insane.

(17) a. If Bizet and Verdi are the same height, the Verdi is short.
$\rightsquigarrow$ Bizet is short.
b. If Bizet and Verdi are the same height, the Bizet is short.
$\rightsquigarrow$ Verdi is short.

(18) a. If Pete called, he won.
$\rightsquigarrow$ Pete had the better hand.
b. If Pete called, his hand was better.
$\rightsquigarrow$ Pete knew Mr. Stone's hand.

Gettier Cases and Truthmakers

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1 The Gettier problem

Traditional goal of epistemology: identify a set of necessary and sufficient conditions for knowing that P (i.e., define *propositional* knowledge).

Traditional answer: knowledge is justified true belief (JTB).

More specifically, S *knows* that P iff

- P is true,
- S believes that P, and
- S is justified in believing that P.

In three of the most influential pages of 20th-century philosophy, Gettier (1963) presents counterexamples to the sufficiency of these conditions (his examples don't threaten their necessity).

Coins. Smith and Jones have applied for a certain job. Smith has been told by the company president that Jones will get the job, and Smith has counted ten coins in Jones pocket a few minutes ago, so Smith justifiably believes the following:

(1) Jones is the man who will get the job, and Jones has ten coins in his pocket.

(Background assumption: "it is possible for a person to be justified in believing a proposition that is in fact false" (p. 121).)

Suppose that Smith infers (2) on the basis of (1) and so comes to justifiably believe this entailment:

(2) The man who will get the job has ten coins in his pocket.

(Background assumption: "for any proposition P, if S is justified in believing P, and P entails Q, and S deduces Q from P and accepts Q as a result of this deduction, then S is justified in believing Q" (p. 121) (i.e., single-premise closure for justified belief).)

Now the punchline: Smith will actually get the job, and unbeknownst to him, he has ten coins in his pocket. Intuitively, Smith has a justified true belief that (2) is true, but he doesn't know that (2) is true.

> But it is equally clear that Smith does not *know* that [(2)] is true; for [(2)] is true in virtue of the number of coins in Smith's pocket, while Smith does not know how many coins are in Smith's pocket, and bases his belief in [(2)] on a count of the coins in Jones's pocket, whom he falsely believes to be the man who will get the job. (p. 122)

Ford. Smith has strong evidence for the following claim:

(3) Jones owns a Ford.

Though Smith is ignorant of his friend Brown's current whereabouts, Smith competently deduces the following from (3):

(4) Either Jones owns a Ford, or Brown is in Barcelona.

In fact, Jones doesn't own a Ford but Brown is in Barcelona. So, again, Smith has a justified true belief that doesn't constitute knowledge.

2 Bringing in truthmakers

One line of intuitive response to Gettier cases is to appeal to the realized fact or facts that make true the target claim and argue that the justified true belief is not connected to these actual truthmakers in an appropriate way (in both of Gettier's own cases, the justified true belief is connected to different non-actual truthmakers for the target claim).

Gettier himself suggests something like this in the above quoted passage. The claim (2) is true "in virtue of" the fact that Smith has ten coins in his pocket (and perhaps also the fact that Smith will get the job, though we might instead think of this as a precondition or enabling condition for Smith having ten coins to be a truthmaker; compare the totality facts needed for quantified statements in Fine and Yablo). However, Smith's justified belief isn't "based" on his evidence for *this* fact but rather on his evidence for the different fact that Jones has ten coins in his pocket (which would be an actual truthmaker for (2) had Jones gotten the job).

Likewise, the actual truthmaker for (4) is Brown's being in Barcelona. However, Smith's justified belief isn't based on evidence for this fact but rather on his evidence that Jones owns a Ford (which is a non-actualized truthmaker for (4)).

This all seems rather promising, but the response remains pretty vague. What is it for one's belief to be appropriately connected to an actual truthmaker so as to constitute knowledge of the claim made true?

Interestingly, Goldman explicitly mentions truthmakers at the start of his classic paper "A Causal Theory of Knowing" (1967), where he proposes that the relevant connections are *causal*:

> Notice that what *makes* [(4)] true is the fact that Brown is in Barcelona, but that this fact has nothing to do with Smith's believing [that (4) is true]. That is, there is no *causal* connection between the fact that Brown is in Barcelona and Smith's believing [that (4) is true]. If Smith had come to believe [that (4) is true] by reading a letter from Brown postmarked in Barcelona, then we might say that Smith knew [that (4) is true]. Alternatively, if Jones did own a Ford, and his owning the Ford was manifested by his offer of a ride to Smith, and this in turn resulted in Smith's believing [that (4) is true], then we would say that Smith knew [that (4) is true]. Thus, one thing that seems to be missing in this example is a causal connection between the fact that makes [(4)] true (or simply: the fact that [that (4)) and Smith's belief of [the truth of (4)]. (p. 358)

3 Kratzer

Kratzer's (2002) semantics for *know* is in the same vein. She considers the following Gettier-style case from Russell's (1912) *Problems of Philosophy*:

> If a man believes that the late Prime Minister's name began with a B, he believes what is true, since the late Prime Minister's last name was Sir Henry Campbell Bannerman. But if he believes that Mr. Balfour was the late Prime Minister, he will still believe that the late Prime Minister's last name began with a B, yet this belief though true, would not be thought to constitute knowledge.

As with Gettier's examples, there is failure of knowledge because belief in (5) isn't based on evidence for the actual truthmaker that Bannerman was the late PM (or we might want to say that the actual truthmaker is that Bannerman's name begins with a 'B', where the fact that Bannerman was the late PM is a precondition for Bannerman's name beginning with a 'B' being a truthmaker for (5)):

(5) The late Prime Minister's name began with a 'B'.

Kratzer contrasts her account with Goldman's:

- GOLDMAN: S knows P iff the fact P is causally connected in an 'appropriate' way with S's believing P.
- KRATZER: S knows P iff S believes P *de re* of some (actual) fact exemplifying P.

Immediate questions:

- What is it to believe a proposition "de re" of some fact?
- What is it for a fact to "exemplify" a proposition?

Kratzer doesn't say very much about belief *de re*. She does say that "for *de re* beliefs to be possible, some causal connection between believers and the *res* of their beliefs is required" (p. 658). So like Goldman, she takes herself to be offering a causal account.

Kratzer sharpens the notion of a fact *exemplifying* a proposition within a situation semantic framework.

Kratzer-style situation semantics in a nutshell:

- Background ontology: set S of possible situations.
- A subset of S is singled out as the set of possible individuals.
- Situations stand in part-whole relations to each other: $\leq$ is a partial order on S and each s is $\leq$-related to a unique maximal element w_s (the world of s).
- Situations can be related across worlds via a Lewisian counterpart relation.
- Propositions are sets of possible situations (i.e., they are properties of situations).
- P is true in situation s iff $s \in P$.
- P can be neither true nor false in s.
- Logical relations depend only on the worlds in which propositions are true to keep the logic classical.

Within this framework, Kratzer defines exemplification as follows (p. 660):

- If s is a possible situation and P a proposition, then s is a **fact exemplifying** P iff for all s' such that $s' \leq s$ and P is not true in s', there is an s'' such that $s' \leq s'' \leq s$, and s'' is a minimal situation in which P is true. (A minimal situation in which P is true is a situation that has no proper parts in which P is true.)

N.B. It seems a lot easier to start with a notion of exact verification and go from there. The infinite stars example also seems very worrisome despite what Kratzer says about it.

Anyways, back to knowledge. Kratzer recognizes that the account so far is problematic. Just as simply saying that knowledge requires belief causally connected to an actual truthmaker leaves much to desired, simply saying that knowledge requires belief *de re* of an actual truthmaker is insufficient. Kratzer brings out the trouble in a somewhat oblique way. Consider:

(6) A child was born yesterday.

This seems easy to know even if one doesn't have a *de re* belief about any particular fact exemplifying the proposition P that is true in any situation located within yesterday in which a child was born. However, Kratzer suggests that (6) can be interpreted as a "thetic statement" that expresses a different proposition P' that is true only in entire worlds. Since we are well acquainted with the world we live in, we can have a *de re* belief of a fact exemplifying the worldly proposition P'.

But now knowledge seems too easy. Can I know (7) if I just so happen to be right?

(7) 2001 children were born yesterday.

We need a justification or reliability condition (this can be brought out more simply and directly by considering fake barn scenarios). Following Goldman (1976), Kratzer states this in terms of *relevant alternatives*. Her final analysis:

S *knows* that P iff

- There is a fact f that exemplifies P,
- S believes P *de re* of f, and
- S can rule out relevant alternatives of f that do not exemplify P.

Note that it is the second condition that is clearly violated in Gettier cases, while the third is (potentially) violated in fake barn cases (this can explain why our judgments about Gettier cases are clearer than our judgments in fake barn cases).

4 Yablo

In §7.4 of *Aboutness*, Yablo ties together truthmakers, Gettier cases, and closure for knowledge.

(8) I have a hand. So, here are physical objects.

(9) I am sitting by the fire. So, I am not a bodiless BIV.

(10) That is a zebra. So, it isn't a cleverly disguised mule.

On Yablo's analysis, the heavyweight consequences are ampliative, not truth-conditionally, but because of their aboutness properties—they change the subject by raising additional issues. The heavyweight consequences aren't *part* of the lightweight premises—they have new ways of being true and/or new ways of being false. For instance, truthmakers for not being a cleverly disguised mule are being a zebra, or unpainted mule, or a lion, or a toaster.

This can make the consequence epistemically vulnerable:

> This is relevant to knowledge insofar as each new disjunct is a new opportunity to believe Q for the wrong reasons. You know that you turned the stove off (P), by virtue of remembering the event. What about the dogmatic implication Q that counterevidence is misleading? There were ten witnesses, let's suppose, and the counterevidence is drawn from their reports. One way for Q to be true is for the first witness to testify against you. Another way for Q to be true is for the first two witnesses to testify against you. And so on. You have got to suppose that the number is small, since as it grows so does the likelihood you are misremembering. You cannot afford to be neutral about how Q is true, since if the story is too fantastic you should not be believing that P. As we know from Gettier, though, mistakes on this score can be knowledge-destroying (Gettier 1963). You are right to regard Q as true, but, if you are sufficiently confused about how it is true—about how things stand with respect to its subject matter—then you don't know that Q. Your evident vulnerability to failing to know in this way—through, as George W. Bush might put it, misunderestimating the counterevidence—may inhibit you even from believing Q, which poses a further threat to knowledge. (p. 118-9)