Composing Menus

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Abstract: I explore an unorthodox perspective on the logical foundations of English on which speakers apply their logical competence by building and composing alternative sets, or 'menus', of entities or states throughout the grammar. The logical connectives are 'menu constructors': conjunction is a collective operation for putting combinations of items 'on' a menu, disjunction contributes nondeterminism or choice between items, while negation renders items 'off menu' by introducing negative entities or states. The system allows for determiner phrases to be interpreted uniformly in a lower type as menus compiled of positive, negative, or hybrid entities, rather than in the higher-order type of generalized quantifiers. Through a new compositional method, the negation contributed by a non-positive entity is able to pass through a semantic derivation in a well-behaved manner. This approach enables a "non-Boolean" collective treatment of sentences involving determiner phrase conjunctions with non-upward entailing conjuncts, which have previously been considered one of the toughest challenges for the collective theory.

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1 From truthmaker to menu semantics

The logical foundations of English and other natural languages are widely believed to have an essentially truth-theoretic character, with the meanings of logical expressions oriented towards deriving the truth values of sentences. Logical words such as *not*, *and*, and *or* (and their correlates in other languages) are commonly interpreted as type-polymorphic operators whose application to semantic values in different types extends the corresponding Boolean truth functions from classical sentential logic (von Stechow 1974; Keenan & Faltz 1978, 1985; Gazdar 1980; Partee & Rooth 1983). Within generalized quantifier theory, quantificational phrases headed by determiners such as *every*, *some*, and *no* function to sift or separate the denotations of verb phrases into those they can combine with to form true sentences and those that lead to falsity (Montague 1973; Barwise & Cooper 1981).

In this article, I investigate and contribute to the case for an alternative perspective on the logic of natural language, one that shifts the focus away from the truth value to what I will call the 'menu', to borrow a metaphor from Kit Fine (2017a). On this perspective, speakers apply their logical competence by constructing and composing alternative sets, or menus, of different items throughout the grammar—determiner phrases of all stripes signify menus of entities, while verb phrases signify menus of states, and so forth. Oddly enough, this truth value-displacing perspective emerged from my attempts to develop a compositional version of 'truthmaker' or 'exact' semantics. In unilateral systems of truthmaker semantics, the proposition expressed by a sentence is a set of *partial* and *exact* verifying states, a menu of *wholly* relevant ways for the sentence to be made true (Fine 2017a; Bernard & Champollion 2024; Champollion & Bernard 2024).[1] In my typed exact semantics—or 'menu semantics', if you will—composing the

[1] 'Truthmaker semantics' refers to a family of semantic theories that are distinguishable both from traditional possible worlds semantics, which relies on *complete* verifying states (i.e., possible worlds), as well as from a crowd of closely related *inexact* semantic frameworks that require only partial relevance between verifiers and the statements they verify, including 'situation semantics' (Barwise & Perry 1983; Barwise 1989; Kratzer 1989, 2002, 2020; Muskens 1995), 'inquisitive semantics' (Ciardelli, Groenendijk, and Roelofsen 2013, 2018), and 'possibility semantics' (Holliday 2021), to name a few. In this paper, I develop a 'unilateral' system of 'recursive' truthmaker semantics. I will not have space to provide a detailed comparison with the 'reductive' approach discussed in Yablo (2014), which posits minimal models as truthmakers, and will only briefly touch on 'bilateral' systems of truthmaker semantics that assign truthmaking and falsemaking states to sentences separately, despite the popularity of this double-entry approach in the truthmaker semantics literature (Van Fraassen 1969; Fine 2017a,b,c).

menus of entities and states contributed by subsentential constituents can determine these truthmaking contents of sentences.[2]

I regard the logical connectives as 'menu constructors', a conception motivated in part by striking parallels between the truthmaker semantic treatment of conjunction and disjunction at the sentential level and related proposals on noun phrase coordination in the linguistics literature. Conjunction is a collective operation for putting combinations of items 'on' a menu. The signature of truthmaker semantics is sometimes thought to be its clause for sentential conjunction, which captures the idea that to be wholly relevant to the truth of a conjunction, a state must make each of the conjuncts true and contain no extraneous material. Given a mereological fusion operation $\sqcup$ defined over the space of truthmaking states, a truthmaker for a sentential conjunction is a fusion of truthmakers for the conjuncts:

(1) $$\begin{aligned} & [\![\textsf{Agrippina sang and Caesar danced}]\!] \\ = \; & \{s \sqcup t : s \in [\![\textsf{Agrippina sang}]\!] \wedge t \in [\![\textsf{Caesar danced}]\!]\} \end{aligned}$$

At the noun phrase level, the collective treatment of conjunction has a longer history, often traced back to influential work in the 1970s and 1980s by Massey (1976), Link (1983, 1984), and Hoeksema (1983, 1988), who introduced mereological structure in the domain of entities, allowing individual conjunctions to denote plural fusions of entities (in my theory, such conjunctions denote a singleton set containing a plural entity):

(2) $$[\![\textsf{Agrippina and Caesar}]\!] \;=\; \{\text{Agrippina} \sqcup \text{Caesar}\}$$

Other semanticists have subsequently argued that *and* has a basic cross-categorial meaning related to plurality formation (Krifka 1990; Lasersohn 1995; Schwarzschild 1996; Heycock & Zamparelli 2005; Schmitt 2013, 2019). While these semanticists have primarily focused on explaining collectivity effects in natural language, and truthmaker semantics has been developed mainly by logicians and philosophers in the business of providing new model theories for various sentential logics and analyzing concepts like partial content, subject matter, and logical subtraction (Van Fraassen 1969; Fine 2014, 2016, 2017a,b,c; Yablo 2014; Jago 2020), this article seeks to unify these different threads from the linguistics and philosophy communities under a single semantic theory.

Disjunction may be thought of as contributing nondeterminism or

[2]Throughout this paper, I use 'state' as a general cover term to refer to events, processes, or circumstances, which linguists often collectively call 'eventualities' (following Bach 1986a). My compositional system of menu semantics may be regarded as a compositional version of event semantics.

choice between items on a menu. In recursive truthmaker semantics, a truthmaker for a sentential disjunction is a truthmaker for one of the disjuncts (Fine 2017c also introduces an "inclusive" form of the semantics where the menu of truthmakers for a disjunction also includes truthmakers for the corresponding conjunction):

(3)
$$\begin{aligned} & [\![\textsf{Agrippina sang or Caesar danced}]\!] \\ = \ & \{s : s \in [\![\textsf{Agrippina sang}]\!] \vee s \in [\![\textsf{Caesar danced}]\!]\} \end{aligned}$$

The idea that disjunction introduces a choice among different alternatives also underlies 'Hamblin-style' or 'alternative semantic' treatments of disjunction (after Hamblin 1973), which have been developed to account for a range of natural language phenomena, including the semantics of indefinites, questions, the logical properties of counterfactual conditionals, and more (see also the related treatment of disjunction in 'inquisitive semantics' (Ciardelli, Groenendijk, and Roelofsen 2013, 2018)). For instance, in Alonso-Ovalle's influential work, an individual disjunction denotes an alternative set whose members are the referents of the disjuncts (Alonso-Ovalle 2006):

(4)
$$[\![\textsf{Agrippina or Caesar}]\!] \quad = \quad \{\text{Agrippina}, \text{Caesar}\}$$

On my semantics, disjunction has the cross-categorial meaning of set union, and this subsumes both truthmaker semantics for sentential disjunctions and alternative semantics for individual disjunctions.

A major challenge for this unified theory of coordination is its integration with negation, in ways I will discuss. In keeping with the menu metaphor, my solution to these difficulties is to treat negation as an operation for rendering items 'off menu'. Inspired by some informal suggestions from Fine on how to "de-sententialize" truthmaker semantics to handle denoting expressions, I unpack this 'off menu' status with recourse to an entity domain that includes both positive entities like Caesar and negative entities like ¬Caesar (pronounced as 'negative Caesar' or 'anti-Caesar'), as well as their fusions (Fine 2017a, Section 4; see also the 'shadow theory' of Akiba 2009, 2015). Additionally, I employ a state domain wherein positive states, such as Caesar crossing the Rubicon, have corresponding negative states as their 'orthogonal counterparts'. Applying negation to the denotation of a determiner phrase introduces negative entities ('entity negation'), while applying it to the value of a verbal or other predicative projection introduces negative states ('event negation'). As a preview of what is to come, note that the first example includes the negative individual anti-Caesar, while the second includes negative states:

(5) $$\begin{aligned} [\![\textsf{Agrippina but not Caesar}]\!] &= \{\text{Agrippina} \sqcup \neg\text{Caesar}\} \\ [\![\textsf{Agrippina didn't sing}]\!] &= \{\textstyle\bigsqcup\{\neg s : s \in [\![\textsf{Agrippina sang}]\!]\}\} \end{aligned}$$

The introduction of negative entities into compositional semantics further allows for the possibility of a non-Montagovian theory of 'quantification' on which determiner phrases are interpreted uniformly in a lower type as menus of entities rather than in the higher-order type of generalized quantifiers or property sets (as in the tradition of Montague 1973; Barwise & Cooper 1981):

(6) $$\begin{aligned} [\![\textsf{a/some magician}]\!] &= \{\text{Houdini}, \text{Presto}, ...\} \\ [\![\textsf{no magician}]\!] &= \{\neg\text{Houdini} \sqcup \neg\text{Presto} \sqcup ...\} \\ [\![\textsf{less than two magi}]\!] &= \{\neg(\text{magus1} \sqcup \text{magus2}) \sqcup \\ &\quad \neg(\text{magus2} \sqcup \text{magus3}) \sqcup ... \\ &\quad \neg(\text{magus1} \sqcup \text{magus2} \sqcup \text{magus3}) \sqcup ...\} \end{aligned}$$

This entity-style treatment builds on previous work on indefinites and indeterminate pronouns by Kratzer & Shimoyama (2002), Menéndez-Benito (2005), and Charlow (2014, 2020), with negative entities enabling its extension to negative quantifiers and, more tentatively (for reasons I discuss later), to modified numerical phrases.

While the hypothesis that English speakers represent entities and states as positively or negatively marked affords new analyses of a variety of negative linguistic items and constructions, I recognize that it also raises significant ontological hesitation and theoretical concern. The theory advanced in this article may be seen as a return to what David Lewis, in 'General Semantics' (1970), referred to as the "dark ages of logic" (p. 52)—presumably alluding to Russell's early theory in *Principles of Mathematics* (1903) on which quantificational phrases denote strange and unusual entities, a theory later discarded for Russell's more influential account in 'On Denoting' (1905) on which quantificational phrases do not themselves have any meaning of their own. Half a century ago, Peter Geach (1962) and Peter Strawson (1974), in writing on the subject-predicate distinction, also presented arguments aimed at demonstrating the logical incoherence of negative names or subject terms, and their arguments anticipate some of the difficulties involved in getting non-positive entities to compose properly.

Nevertheless, my hope, at a minimum, is to convince you that negative entities can serve as respectable, law-abiding citizens within a semantics for a fragment of English that is consistent and capable of providing appropriate truthmaking conditions for sentences in this fragment—although such a theory must be crafted with some care.[3] I argue that there are relatively innocent ways of understanding negative entities that do not entail a commitment to a mysterious realm of shadowy creatures or any other metaphysical extravagance. I demonstrate how the challenges presented by Geach and Strawson can be overcome, and I introduce a new compositional method that enables non-positive entities to pass their negation through semantic derivation in a well-behaved manner. Of particular importance to the debate between the collective "non-Boolean" theory of conjunction based on plurality formation versus the traditional intersective "Boolean" theory based on logical conjunction, I show how a collective conjunction can be combined with my novel semantics for negation to interpret determiner phrase conjunctions with non-upward entailing conjuncts, which have previously been considered one of the toughest challenges for the collective theory. It seems difficult, if not impossible, to provide a satisfying collective treatment of examples such as the following:

(7) Agrippina and nobody else danced.

Champollion (2016), for one, argues that such coordination structures require an intersective conjunction operating on generalized quantifier denotations to determine appropriate truth conditions. Against this, I demonstrate how a non-Boolean theorist about conjunction can provide a relatively straightforward analysis of such cases by helping themself to negative entities. This makes it possible to interpret a coordinated subject like *Aggripina and nobody else* as a menu containing the hybrid or 'mixed polarity' entity Aggripina$\sqcup\neg$Caesar$\sqcup\neg$Brutus$\sqcup$..., which has Aggripina and the negative counterparts of everybody else as its atomic parts. Through composition within my system of truthmaker semantics, this entity menu can produce the correct kind of truthmaker for the example, consisting of a positive state of Aggripina dancing fused with negative states of everyone except Aggripina not dancing.

This paper is organized as follows. In Section 2, I lay out in more detail the challenge that examples like (7) pose for the collective theory of conjunction, as proposing a solution to this challenge is one of the main applied contributions of the paper. In Section 3, I introduce the model theory for my compositional menu semantics and the Neo-Davidsonian

[3] As a reviewer recognizes, the introduction of negative entities is not wedded to truthmaker semantics. While I deploy negative entities to meet challenges arising from the interaction of negation and collective conjunction within a compositional generalization of truthmaker semantics, they could be integrated into other semantic frameworks, provided that analogous compositional rules are introduced to ensure their negative contributions are properly managed.

framework in which this semantics will be developed. Sections 4 and 5 present my menu semantics for coordination and negation, respectively. Section 6 presents my menu semantics for basic quantifiers. In Section 7, I explore the issue of how to compose non-positive entities. After examining Geach and Strawson's arguments, I propose a novel compositional method that allows the negation contributed by negative entities to exhibit the correct scope-taking behavior. In Section 8, I apply the resulting theory to the challenge cases for collective conjunction and provide a tentative treatment of some non-upward entailing numerical phrases. In Section 9, I compare my semantics for negation to earlier approaches by Fine and by Champollion and Bernard, which are based on the notion of 'exclusion' between states, and I propose a way to incorporate an exclusion relation into my system. Section 10 concludes and suggests directions for further research.

2 Troubles for collective conjunction

One central theme of this article is the challenge of integrating a collective semantics for conjunction with negation, so to set the stage, I would like to begin by outlining the ongoing debate between "Boolean" and "non-Boolean" theories of conjunction, along with specific difficulties that negation poses for the non-Boolean theory.

The traditional Boolean theory is supported by intersective or distributive uses of *and*, as demonstrated by the examples in (8), which are standardly accounted for by assuming that *and* denotes an operator that, when applied to truth values and other semantic values, recursively extends the corresponding truth function from formal logic (von Stechow 1974; Keenan & Faltz 1978, 1985; Gazdar 1980; Partee & Rooth 1983; among many others):

(8) a. Alfonso and Beatrice (each) ate a piece of pie.
    b. Claribel sang and danced.
    c. Claribel sang and Donatello danced.

On the other hand, the word *and* can also be used collectively or nondistributively, as shown by sentences with collective predicates like those in (9), and by sentences with 'cumulative' or 'cross-product' readings like those in (10), where, for instance, the cumulative reading of (10-a) is verified by a state of either Pearl or Rose feeding one cat while the other feeds the second, and (10-b) is verified by a state where, for example, five animals crow while the remaining five bark:

(9) a. Beatrice and Claribel met in the town square.
b. A soprano and a tenor sang a duet.
c. Ten men and women got married today in San Pietro. (Heycock & Zamparelli 2005)

(10) a. Pearl and Rose fed Fluffy and Marmalade.
b. The ten animals are crowing and barking. (Schmitt 2019, adapted from Krifka 1990)

Many authors have argued that, in such cases, English conjunction functions not like logical conjunction, but rather as an operator for forming plural entities (Massey 1976; Link 1983, 1984; Hoeksema 1983, 1988; Krifka 1990; Lasersohn 1995; Schwarzschild 1996; Heycock & Zamparelli 2005; Schmitt 2013, 2019; among others).

Research on the semantics of conjunction has tried to determine whether the basic meaning of *and* is associated with logical conjunction or plurality formation. Although Partee & Rooth, Link, and Hoeksema originally suggested that *and* is lexically ambiguous between intersective and collective meanings, I set this possibility aside in this article. I agree with Kripke (1977) and others that we should only posit ambiguities when really forced to, and, moreover, an ambiguity theory would leave unexplained why most, if not all, languages are like English in using the same word or marker to conjoin expressions of a specific syntactic category, regardless of whether the intended meaning of the conjunction is intersective or collective (Winter 1996, citing Payne 1985).

The situation is sometimes characterized as a stalemate. Boolean theorists can easily account for distributive examples like those in (8), but they have had to posit silent operators involving set minimization and choice functions to handle nondistributive examples like those in (9) (Winter 1996, 2001; Champollion 2016). Non-Boolean theorists, on the other hand, have an easier time with the examples in (9) and (10), as well as with other coordination structures that give rise to collectivity effects, but they have had to appeal to a silent distributivity operator to derive the intended reading of (8-a) (Link 1987), and furthermore, sentential and other coordinations beyond the level of the determiner or noun phrase have been thought to be less amenable to a collective approach (see the concluding remarks of Heycock & Zamparelli 2005).

For what it's worth, I think that, in our current state of research, the non-Boolean theorist has the edge. Although the collective theory was originally motivated by collective predication of individual conjunctions, truthmaker semanticists have since provided a rigorous formal and philosophical foundation for the collective treatment of sentential conjunctions, and they have begun to explore the linguistic applicability of this approach (Fine 2017c; Yablo 2016; Moltmann 2020, 2021; Güngör 2023; Champollion & Bernard 2024; among others). With this research in mind, I think the prospects for a unified collective semantics for conjunction are now stronger than ever. Furthermore, on the empirical side, recent cross-linguistic work by Flor et al. (2017a,b) using the TerraLing database has shown that, assuming that covert operations in English should be expected to show up overtly in other languages, there is evidence that predicate-level distributivity operators are available for deriving distributive readings, however there is no evidence that the silent operators posited by Boolean theorists are at work in deriving collective readings. Cross-linguistically, the basic lexical meaning of conjunction seems to be collective.

This being said, my intention in this article is not to conclusively establish the collective theory as superior to the intersective theory. Rather, my more modest objective is to go some way towards addressing some of the outstanding challenges faced by the collective theory, particularly its interaction with negation and related linguistic items. Despite the collective theory's success in predicting the plurality forming behavior of *and* in nondistributive environments, the intersective theory may still be thought to hold a decisive advantage: to my knowledge, collective theorists have yet to provide a fully satisfactory semantics for the determiner phrase that harmonizes with their approach. Arguably the most difficult cases are conjunctions involving non-upward entailing quantifiers, as in the following examples:

(11) a. John and nobody else smiled.
b. John and between one and three women smiled.
c. John and an odd number of women smiled.

Since these examples pose no special problem for intersective conjunction, which can operate on generalized quantifier denotations to determine the correct truth conditions, they suggest that, absent a viable collective account, we should adhere to the intersective theory.

Champollion (2016) discusses the difficulty for Heycock & Zamparelli's (2005) influential collective account. Heycock & Zamparelli adopt a set-based approach to plurality in the entity domain, where atomic individuals are represented by singleton sets such as $\{\text{John}\}$, and plural entities, like the sum of $\{\text{John}\}$ and $\{\text{Mary}\}$, are represented by non-singleton sets such as $\{\text{John}, \text{Mary}\}$. Common nouns and verb phrases denote higher-order sets of these entities. Within this framework, Heycock & Zamparelli interpret conjunction as a cross-product operation that combines two sets of sets, forming a new set in which each member is the union of elements drawn from each operand (see also Landman's 2004 'Sum Pairing' for a similar proposal):

(12) **Heycock & Zamparelli's *and* as 'Set Product'**
a. $\mathrm{SP}(S_1, ..., S_n) \quad := \quad \{X_1 \cup ... \cup X_n : X_1 \in S_1, ..., X_n \in S_n\}$
b. $[\![\textsf{and}]\!] \quad = \quad \lambda S \lambda S'.\mathrm{SP}(S, S')$

Assume that names denote individuals, which can be wrapped within a second set using a type-shifter ↑ that lifts a semantic value into a singleton set containing this value. The denotation of an individual conjunction formed by Set Product is a (singleton of) a Linkian plurality:

(13)
$$\begin{aligned} [\![\textsf{John and Mary}]\!] &= [\![\textsf{and}]\!](\uparrow [\![\textsf{John}]\!])(\uparrow [\![\textsf{Mary}]\!]) \\ &= \mathrm{SP}(\{\{\mathrm{John}\}\}, \{\{\mathrm{Mary}\}\}) \\ &= \{\{\mathrm{John}, \mathrm{Mary}\}\} \end{aligned}$$

However, as Heycock & Zamparelli themselves acknowledge, Set Product does not straighforwardly extend to examples involving quantifiers. Assume now that *John* and *nobody (except perhaps John)* denote, or can be type-lifted to, the following generalized quantifiers:

(14)
$$\begin{aligned} [\![\textsf{John}]\!] &= \lambda P.P(\{\mathrm{John}\}) \\ [\![\textsf{nobody (except perhaps John)}]\!] &= \lambda P.\forall x(x \neq \{\mathrm{John}\} \to \neg P(x)) \end{aligned}$$

Applying Set Product to these denotations (in their corresponding set form), the conjunction *John and nobody else* denotes a set that includes any union of a set containing (the singleton of) John with a set that doesn't contain (the singleton of) anyone else, except possibly John. Since this union may still contain (the singletons of) individuals other than John, this is not what we want for evaluating (11-a).

This difficulty, along with the challenges posed by the other examples in (11) with non-upward entailing constituents, leads Champollion to conclude that the collective theory of conjunction is untenable, especially if we want to continue working with generalized quantifiers:

> The hardest nut to crack for anyone wishing to pursue the collective theory is probably coordination of non-upward entailing quantifiers such as *John and nobody else* or *John and an odd number of women.* Not only do Heycock & Zamparelli (2005) not give a satisfying account of these conjunctions, it also does not seem easy to give one under any approach that takes the basic meaning of *and* to be collective. For this reason alone, it seems preferable to make the intersective theory work if one is interested in

> using generalized quantifier denotations for at least some non-upward entailing noun phrases. (p. 612)

We seem to have two options: either maintain the Boolean theory of conjunction, or accept that generalized quantifier theory cannot be applied to all determiner phrases, despite its being regarded as one of the major success stories in formal semantics, and with it being far from clear how an alternative account of negative quantifiers could work.

I do not think there is an easy way out of this choice point. Heycock & Zamparelli suggest that examples like (11-a) with *no* might be dealt with by decomposing this negative quantifier into an indefinite within the scope of a sentential negation operator that is free to take a higher scope than the rest (Klima 1964; Ladusaw 1992):

(15) $\neg[_{\text{TP}}[_{\&\text{P}}[_{\text{DP}}$John] and $[_{\text{DP}}$somebody else]] smiled].

However, as Champollion notes, this scope-splitting analysis incorrectly predicts that (11-a) would mean the same as *It is not the case that John and somebody else smiled* (see also Landman 2004, Section 8.5).

Alternatively, one could argue that the examples in (11) do not involve the coordination of determiner phrases but are rather cases of backward 'Conjunction Reduction' (CR) or 'Right Node Raising' (RNR; Postal 1974; Wilder 2018). If the verb in each sentence is shared by two parallel phrases within a sentential coordinate structure—whether through backward ellipsis, movement, multiple dominance, or another mechanism—then there is no puzzle of subsentential coordination to worry about (these examples could instead be interpreted using the sentential collective conjunction from truthmaker semantics):

(16) $[_{\&\text{P}}[_{\text{TP}}$John ~~smiled~~] and $[_{\text{TP}}$nobody else smiled]].

For several reasons, though, I agree with Champollion that it is preferable to pursue a direct analysis rather than a CR analysis. To be clear, I am not claiming that cases of apparent subject or object DP conjunctions with non-upward entailing constituents can *never* involve covert underlying syntax that supports a sentential conjunction analysis (see Hirsch 2011 for arguments involving the distribution of adverbials and split-scope phenomena that CR structures are often available for surface DP conjunctions). I only wish to argue against the claim that *and* must be uniformly interpreted as sentential in all relevant cases.

A preliminary consideration is that, in typical examples used to motivate CR analyses in the syntax literature, such as (17) and (18), sharing is required to maintain standard assumptions about constituency structure, namely, that a conjunction and its individual conjuncts are syntactic constituents:

(17) Alfonso read and Claribel burned the letter.

(18) Alfonso is happy that Beatrice, and upset that Claribel, are coming to the wedding.

By contrast, expressions like *John and nobody else* and *John and an odd number of women* are constituents according to standard diagnostics—they can serve as fragment answers, be replaced by pro forms, appear as the initial element in wh-cleft constructions, and so forth—therefore, there is no impetus from constituency for a backward CR-type analysis as sentential coordination.

Examples like those in (11) also do not exhibit the typical prosodic profile of Right Node Raising. Sentences like (17) and (18) are pronounced with a distinctive L+H*L− intonational contour at the right edge of the non-shared material in each conjunct—marked by a sudden rise to a high pitch accent followed by a sharp fall and often a phonological phrase break, which Selkirk (2002) calls the "Duncecap" pattern—whereas the examples in (11) lack this intonation.

More importantly, coordinations with non-upward entailing quantifiers can appear within sentences that do not readily admit sentence coordination paraphrases, particularly those with collective predicates like *gather*, which require semantically plural arguments:

(19) a. John and between one and three women gathered outside.
b. #John gathered outside and between one and three women gathered outside.

(20) a. John and none of the women except Mary gathered outside.[4]
b. #John gathered outside and none of the women except Mary gathered outside.

Cumulative readings for the following sentences also seem underivable through CR, which would increase the number of muffins eaten by John from fewer than five muffins in the (a)-sentences to five in the (b)-sentences:

(21) a. John and between one and three women ate five muffins (between them).
b. John ate five muffins and between one and three women

[4] I am grateful to an anonymous reviewer for suggesting this kind of example.

ate five muffins.

(22) a. John and none of the women except Mary ate five muffins (between them).
b. John ate five muffins and none of the women except Mary ate five muffins.

Hirsch & Sauerland (2019) propose the distribution of sentential *also* as a diagnostic for probing underlying syntactic structure. Since *also* requires a sentential host, they argue that examples like (23) should be analyzed as covert sentential coordinations (obscured by RNR):

(23) John, Bill, and also Mary met in the yard.

On the other hand, *also* cannot adjoin within other collective conjuncts, indicating they cannot be re-analyzed with hidden sentential structure:[5]

(24) #John and also Mary met in the yard.

Notably, *also* is unavailable in (19-a) and (20-a), supporting the claim that these examples do not involve covert sentential nodes:

(25) #John and also between one and three women gathered outside.

(26) #John and also none of the women except Mary gathered outside.

These collective predications therefore challenge the non-Boolean theory of conjunction, as direct analyses are available that utilize Boolean conjunction together with generalized quantifier theory.

'Adversative coordinations' formed with *but* present an additional hard nut for the collective theorist to crack. Vicente (2010) argues that sentences involving "corrective *but*", which have negation in the first conjunct, as in (27), always require clause-level coordination. However, Toosarvandani (2013) rejects this clause-only hypothesis and presents a battery of syntactic arguments that corrective *but* is a cross-categorial coordinator capable of combining negated and unnegated DP constituents:

(27) a. Lysander loves not Hermia but Helena.
b. Not a mathematician but a physicist discovered the neutron.

[5]Hirsch & Sauerland present the *also* insertion data as a problem for Schein's (2017) proposal to re-analyze collective predication and other collective environments using sentential Boolean conjunction and plural event pronouns. I tend to agree, though I cannot attempt to refute Schein's book-length argument for CR.

Vicente also leaves it open whether subclausal coordination is possible in English examples involving what he terms "counterexpectational *but*", where negation appears in the second conjunct:

(28) a. Hermia but not Helena is coming to the wedding.
b. A mathematician but not a physicist will be on the conference panel.

If Toosarvandani is right about corrective *but*, or if counterexpectational *but* can conjoin subclausal constituents, the collective theorist needs to worry about these additional adversative coordinations as well. Here, too, Heycock & Zamparelli's analysis of conjunction as Set Product struggles to deliver the correct results.

As advertised in Section 1, I will try to meet the challenge head-on by showing that deploying negative entities, and rethinking the meaning of determiner phrases in terms of these novel elements, can help the collective theorist to account for *and* and *but* conjunctions with non-upward entailing constituents. By utilizing negative entities in semantic processing, DPs can be interpreted as menus of positive, negative, or hybrid entities, and items on these menus can be fused together through conjunction. Take (27-b), for example. I will assign the indefinite *a physicist* the alternative set or menu containing all physicists within a given quantificational domain, such as Einstein, Chadwick, and so on. The negated indefinite *not a mathematician* will be assigned a menu with a single item, the fusion of all anti-mathematicians from within the domain, that is, $\neg$Hilbert $\sqcup$ $\neg$Ramanujan $\sqcup$ .... Applying collective conjunction to these denotations can generate a new menu containing hybrid elements formed by fusing together a physicist with all the anti-mathematicians, such as Einstein$\sqcup\neg$Hilbert$\sqcup\neg$Ramanujan$\sqcup$.... Finally, using my system's compositional machinery, each of these hybrid elements will determine one of the truthmakers for (27-b), consisting of a state of a specific physicist discovering the neutron fused with negative states of the various mathematicians failing to make this discovery.

3 Semantic foothills

I formally develop the treatment of negative entities using a typed version of truthmaker semantics (Van Fraassen 1969; Fine 2014, 2016, 2017a,b,c; Yablo 2014; Moltmann 2020, 2021; Jago 2020; Bernard & Champollion 2024; Champollion & Bernard 2024). This work is carried out within an intensional system with the following types:

(29) **Types:** e and s are the basic ground types of **entities** and **states**, $\mathsf{a} \to \mathsf{b}$ is the type of a **function** mapping arguments from type a to results of type b, Sa is the type of a **set** of values of type a.

Note that this type hierarchy differs from the type systems in more familiar logics, such as Montague's (1973) higher-order IL or Gallin's (1975) more perspicuous Ty2, in lacking a basic type t of *truth values*. That said, a set type Sa can be put into one-to-one correspondence with the characteristic function $\mathsf{a} \to \mathsf{t}$, so there is a sense in which truth values remain implicitly present in the system.[6]

In my cross-categorial collective semantics for conjunction, I rely on mereological structure in both the entity and state spaces. Following most directly Fine's formal foundations for truthmaker semantics (Fine 2017a,b,c), I assume that the state space is a complete join semilattice, and I make the same assumption regarding the entity space. This means that every subset X of entities or states, including the empty set, has a unique least upper bound, denoted as $\bigsqcup X$, which we can regard as the *sum* or *fusion* of the elements in X. As a special case, I write $x \sqcup y$ for the sum $\bigsqcup\{x, y\}$. When $x \leqslant y$, or equivalently $x \sqcup y = y$, I say that x *is a part of* y, or that y *contains* x. In the case where X is the complete underlying set of entities or states, $\bigsqcup X$ is the *full entity* or *full state* that is greater than or equal to every other element (i.e., $x \leqslant \bigsqcup X$ for each $x \in X$). The entity and state spaces are also bounded from below in that $\bigsqcup \emptyset$ is the *null entity* or *null state* that is less than or equal to every other element (i.e., $\bigsqcup \emptyset \leqslant x$ for each $x \in X$).[7] Within a bounded lattice, an *atom* x is a non-null minimal element that has no proper parts except for the bottom element. When an atom x is part of y, I call x an *atomic part* of y and write $x \leqslant_{\mathrm{AT}} y$. The set of atomic parts of x is $\mathrm{AT}(x) := \{y : y \leqslant_{\mathrm{AT}} x\}$. A bounded lattice is said to be *atomistic* when each of its elements is 'built' from its atomic parts: $x = \bigsqcup \mathrm{AT}(x)$ for all x. I assume that the entity and state spaces are atomistic, though this assumption is relevant here only to my use of atomic individuals as the referents of proper names.

It is less clear what kind of structure can underwrite a cross-categorial semantics for negation that allows for both entity and event negation. One approach inspired by the truthmaker semantics literature is to introduce a binary *exclusion* relation on the entity and state spaces,

[6]Thanks to an anonymous reviewer for prompting me to clarify this point.

[7]Although the incorporation of mereology in the entity domain is common in the linguistics literature on plurality, the null entity is usually left out. A notable exception is Landman (2004).

following work by Fine (2017a) and by Champollion & Bernard (2024), who develop unilateral systems of truthmaker semantics that involve an exclusion relation between states. While the precise details of these systems differ (in ways I discuss later in Section 9), they use the exclusion relation to define a mapping from any set of states to another set of states each member of which precludes every member of the original set from occurring. English *not* is taken to express this function, which Champollion & Bernard call "event negation" and abbreviate as NEG:

(30) $$\begin{aligned} & [\![\textsf{Apollo didn't sing}]\!] \\ = \; & \{s : s \in \mathrm{NEG}([\![\textsf{Apollo sang}]\!])\} \end{aligned}$$

It is important to note that an exclusion relation alone does not distinguish between positive and negative elements. In fact, neither Fine nor Champollion & Bernard assume the existence of intrinsically negative states or events, even though they work with an exclusion relation on the state space. According to them, an excluder and the state or event it excludes needn't be intrinsically distinct from one another in terms of polarity. Fine gives the example of Socrates being Greek excluding his being Roman, where the state of Socrates being Greek and the state of his being Roman are both normal, non-negative states. Champollion & Bernard characterize a "negative event" simply as an event described by a negated sentence, and they allow that a single event, such as one that excludes Mary's leaving, can be considered negative under one description (an event of Mary not leaving) but not negative under another description (an event of Mary staying).

Those who have concerns about the existence of intrinsically negative entities may thus attempt to understand the concept of a 'negative entity' through the exclusion of entities. However, it is not clear that this approach would buy us ontological parsimony. Fine (2017a) suggests that the referent of *not Socrates* is an individual that is present when Socrates is absent, and vice versa (pp. 636–7). But who exactly is this individual that excludes Socrates? Fine does not exclude the possibility of conceptualizing this excluder as an ordinary positive individual, but it is not as if Socrates has a twin that he trades places with. While exclusion may help alleviate concerns about the ontology of event negation, where we can often identify ordinary non-negative states as plausible truthmakers for negated sentences, it does not seem to help much in the case of entity negation, where the excluders of individuals such as Socrates remain mysterious creatures of shadow.

For this reason, the formal and conceptual foundations of my treatment of negation do not rely on exclusion.[8] From a purely formal perspective, my commitment to negative ‘ontology’ may actually seem stronger than that of Fine and Champollion & Bernard: my model theory will distinguish explicitly between positive and negative entities, and between positive and negative states, where this polarization does not depend on the expressions used to describe or denote them. However, from a conceptual standpoint, I would like to avoid ontologically committing myself to negative entities and states. I was initially hesitant to introduce negative individuals into the model theory and experimented with other options. But I have since learned to stop worrying and embrace negative elements in both the entity and state domains, though I understand these items non-metaphysically.

There is a widely held belief that model-theoretic semantics in the tradition of Montague, Lewis, and Creswell carries metaphysical implications, if not for *real* metaphysics, then at least for “natural language metaphysics”, or what we talk as if there is (Bach 1986b; Moltmann 2017). According to this ‘metaphysical perspective’, the various elements and structures of a model provided for the meanings of expressions in a natural language are meant to correspond to objects and relationships in the world as the users of this language take it to be. Now, I certainly do not wish to deny that at least some components in our models for a natural language may be understood in this ontologically-committing way (perhaps we should be ontologically committed to any entities and relations that feature in the truthmakers computed in a semantic derivation).[9] However, I see no reason why every component in a model provided for English must be regarded as an attempt to capture an ontological feature of English metaphysics.

According to an alternative psychological perspective on model-theoretic semantics that I have come to favor, the choice of features to include in a model is primarily an attempt to capture, not something about what the world is like according to the speech habits of a community of language users, but rather something about the internal mental representations and data structures used in natural language processing by members of that community. From this ‘mentalistic perspective’, decisions about the general structure and content of our semantic models are proposals about how we compute meanings. As such, the inclusion of negative entities and states in my domains needn’t carry any metaphysical baggage, nor should they be

[8] An exclusion relation between states can still be incorporated into my system, as I show in Section 9 after much of my analysis of negation has been put in place.

[9] Negative entities play a role in the derivation of truthmakers in my theory, but they do not actually appear within the truthmakers themselves.

viewed as a purely technical device for deriving good results. My basic empirical assumption is that English speakers represent entities and states dichotomously as positively or negatively marked in semantic reasoning. We may, as a first approximation, think of the polarity marking on entities as a way of keeping track of whether individuals are participants or non-participants in states (i.e., whether they are present or absent in states). However, I think the nature of this dichotomous system of representation is revealed by the function it plays within our semantic competence, rather than standing in need of an independent characterization in other terms. While I think there is little harm in reifying negatively marked or tagged entities as elements in the entity domain (and I respond to further objections to negative entities in Section 5), I should add that other ways of encoding polarization could be pursued, such as working with more standard domains in the model theory and then having the compositional semantics itself introduce polarity marking using ordered pairs like $\langle \text{Apollo}, 1\rangle$ and $\langle \text{Ceres}, 0\rangle$.

Formally, I implement the polarization of entities and states by introducing a unary function $\neg$ on both the entity and state domains, where I call $\neg x$ the *orthogonal counterpart* of x. It is assumed that $\neg$ is an involution, meaning that $\neg\neg x = x$ for any x, allowing us to speak of pairs of elements like Apollo and ¬Apollo as *orthogonal* to one another. To construct the polarized spaces needed for my semantics, one can use techniques from the construction of free objects in universal algebra (Birkhoff 1940; Burris & Sankappanavar 1981). Full formal details are provided in an appendix, but for now it is enough to note that the entity space includes: (i) a complete join subsemilattice of *positive entities*, such as Apollo and Bacchus⊔Ceres, as well as (ii) the orthogonal counterparts of these positive entities, such as ¬Apollo and ¬(Bacchus⊔Ceres), which are the *negative entities*, each of which is an atom of the space (except for the null entity, which I take to be both positive and negative), and the remaining elements are (iii) fusions of these positive and negative entities, such as ¬Apollo ⊔ ¬Bacchus and Bacchus ⊔ ¬Ceres. The state space has the same polarity structure.

The domains for higher-order types are defined in the usual way. The domain of values in a function type $\mathsf{a} \to \mathsf{b}$ is the set of all functions from arguments in type a to results in type b. The domain of values in a set type Sa is the power set consisting of every set of values in type a. Unlike the ground-level entity and state spaces, these higher-level spaces are not endowed with their own mereological or polarity structure.

With this model-theoretic framework in place, we can begin developing a compositional semantics for a small fragment of English. I work within a fairly standard off-the-shelf Neo-Davidsonian event semantic framework and make no claims of originality until I turn to providing a menu semantics for connectives and some quantificational expressions in subsequent sections. In my theory, proper names are evaluated as positive atomic individuals in type e, however they can be lifted into the set type Se using the following type shifter, which is assumed to be freely available when needed to avoid type clashes (this type shifter was previously introduced in example (13)):

(31) **Set return**

$$\uparrow \;:=\; \lambda x.\{x\} \qquad\qquad \uparrow :: \mathsf{a} \rightarrow \mathsf{Sa}$$ [10]

(32)
$$[\![\mathsf{Apollo}]\!] = \text{Apollo} \qquad \mathsf{e}$$
$$\uparrow [\![\mathsf{Apollo}]\!] = \{\text{Apollo}\} \qquad \mathsf{Se}$$

I assume that verbs have a Neo-Davidsonian semantics on which they denote sets of positive states (following Carlson 1984; Parsons 1990; Krifka 1992; and much subsequent work):

(33)
$$[\![\mathsf{sing}]\!] = \{s : \text{sing}(s)\} \qquad \mathsf{Ss}$$
$$[\![\mathsf{love}]\!] = \{s : \text{love}(s)\} \qquad \mathsf{Ss}$$

Somewhat less standardly, I extend the Neo-Davidsonian semantics to common nouns by interpreting them as denoting sets of states instead of sets of entities (as suggested by Larson 1998 and Schwarzschild 2009, 2024). For example, being a magician, on this treatment, amounts to participating in a state belonging to the extension of *magician*:

(34)
$$[\![\mathsf{magician}]\!] = \{s : \text{magician}(s)\} \qquad \mathsf{Ss}$$

Another pillar of the Neo-Davidsonian approach is that states are associated with their participants via thematic roles. These roles, such as Agent and Theme, are functions of type $\mathsf{s} \rightarrow \mathsf{e}$ from states to entities. I assume that thematic roles are introduced by syntactic correlates in LF, such as the silent theta-role heads $[\mathsf{Ag}]$ and $[\mathsf{Th}]$ (Kratzer 1996):

(35)
$$[\![[\mathsf{Ag}]]\!] = \lambda e.\{s : \text{Agent}(s) = e\} \qquad \mathsf{e} \rightarrow \mathsf{Ss}$$
$$[\![[\mathsf{Th}]]\!] = \lambda e.\{s : \text{Theme}(s) = e\} \qquad \mathsf{e} \rightarrow \mathsf{Ss}$$

Through Functional Application, theta-role heads can compose with expressions or traces of type e to yield the set of states that are linked to the entity denoted by the e-type argument via the corresponding role. This Ss-type value can be integrated with the Ss-type interpretation of a verbal or other predicative projection through Predicate Modification,

[10] Notation: $\alpha :: \mathsf{a}$ specifies that α is in the type a.

yielding the intersection of these sets of states. In this paper, I adopt the VP-internal subject hypothesis (Koopman & Sportiche 1991), which assumes that subjects are base-generated within a verbal projection and subsequently move out of it, leaving a trace e behind that is bound by an operator λe inserted at the end of the movement (later I provide a rationale for this assumption, particularly in relation to negation).

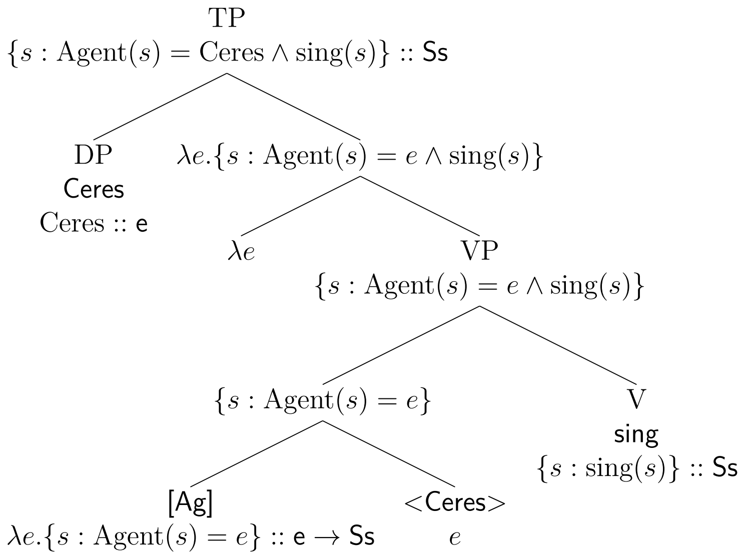

Figure 1: Derivation of Ceres sang

It is worth noting that, although I give English *and* a collective treatment, the predicate modification rule represents a means by which logical conjunction is still incorporated into the system.

Additionally, I must point out that this paper does not delve into matters of tense and aspect. While there are intriguing questions surrounding the integration of tense and aspectual operators into an exact semantic framework, exploring these extensions will have to be reserved for another occasion.

4 Menu semantics for coordination

The next two sections present a 'menu semantics' for the English words *and*, *or*, and *not*. I assume that these connectives can be used to form logical compounds of nominal, predicative, or sentential expressions of type Se or Ss, which denote sets of entities or states. These logically privileged types are referred to as 'menu types':

(36) **Menu types**

$\mathsf{Ma} := \mathsf{Sa}$ for $\mathsf{a} \in \{\mathsf{e}, \mathsf{s}\}$.

My collective rule for *and* treats this coordinator as an operator for placing plural entities or states on a menu:

(37) **Generalized conjunction**

$$[\![\mathsf{and}]\!] = \lambda X \lambda Y.\{x \sqcup y : x \in X, y \in Y\} \qquad \mathsf{Ma} \to \mathsf{Ma} \to \mathsf{Ma}$$

It will be convenient to introduce notation for the pointwise fusion operation applied in this rule, as it will come up again:

(38) Where $X_1, X_2, ...$ are Ma-type sets of entities or states,

$$\begin{aligned} \dot{\bigsqcup}\{X_1, X_2, ...\} &:= X_1 \dot{\sqcup} X_2 \dot{\sqcup} ... \\ &:= \{x_1 \sqcup x_2 \sqcup ... : x_1 \in X_1, x_2 \in X_2, ...\} \end{aligned}$$

The semantic rule for conjunction can now be restated as follows:

(39) $$[\![\mathsf{and}]\!] = \lambda X \lambda Y. \dot{\bigsqcup}\{X, Y\}$$

At the sentential level, this recovers recursive truthmaker semantics. A sentential conjunction specifies a menu of truthmaking states that fuse together truthmakers for each conjunct:

(40) **Conjoining sentences**

$$\begin{aligned} [\![\mathsf{A\ sang\ and\ C\ danced}]\!] &= [\![\mathsf{A\ sang}]\!] \dot{\sqcup} [\![\mathsf{C\ danced}]\!] \\ &= \{s \sqcup t : s \in [\![\mathsf{A\ sang}]\!] \wedge t \in [\![\mathsf{C\ danced}]\!]\} \quad \mathsf{Ms} \end{aligned}$$

Likewise, a conjunction of verb phrases or other subclausal eventuality descriptions denotes a menu of states that fuse together states contributed by each conjunct:

(41) **Conjoining verbs**

$$\begin{aligned} [\![\mathsf{sing\ and\ dance}]\!] &= [\![\mathsf{sing}]\!] \dot{\sqcup} [\![\mathsf{dance}]\!] \\ &= \{s \sqcup t : s \in [\![\mathsf{sing}]\!] \wedge t \in [\![\mathsf{dance}]\!]\} \qquad \mathsf{Ms} \end{aligned}$$

At the individual level, conjunctions introduce plural entities, as in the tradition of Link (1983, 1984, 1998) and Hoeksema (1983, 1988). Note that the semantic rule for conjunction is only defined for set type operands, therefore the referents of the proper names in an individual conjunction must first be transformed into singleton sets using Set Return before this rule can be applied:

(42) **Conjoining names**

$$[\![\mathsf{Apollo\ and\ Ceres}]\!] \;=\; \uparrow [\![\mathsf{Apollo}]\!] \sqcup \uparrow [\![\mathsf{Ceres}]\!] \;=\; \{\text{Apollo} \sqcup \text{Ceres}\} \qquad \mathsf{Me}$$

In sum, the collective rule for conjunction unifies the truthmaker semantic treatment of sentential conjunctions with the Linkian collective approach to individual conjunctions, while also extending to verb phrasal and other predicative conjunctions.

Turning to disjunction, my semantic rule also generalizes the recursive truthmaker semantic treatment by introducing nondeterminism or choice among multiple items on a menu:

(43) **Generalized disjunction**

$$[\![\mathsf{or}]\!] \;=\; \lambda X \lambda Y. X \cup Y \qquad \mathsf{Ma} \to \mathsf{Ma} \to \mathsf{Ma}$$

Sentential disjunctions specify a set of alternative states which verify either of the disjuncts:

(44) **Disjoining sentences**

$$[\![\mathsf{A\ sang\ or\ C\ danced}]\!] \;=\; [\![\mathsf{A\ sang}]\!] \cup [\![\mathsf{C\ danced}]\!] \;=\; \{s : s \in [\![\mathsf{A\ sang}]\!] \vee s \in [\![\mathsf{C\ danced}]\!]\} \qquad \mathsf{Ms}$$

The treatment of verbal and other predicative disjunctions is similar:

(45) **Disjoining verbs**

$$[\![\mathsf{sing\ or\ dance}]\!] \;=\; [\![\mathsf{sing}]\!] \cup [\![\mathsf{dance}]\!] \;=\; \{s : s \in [\![\mathsf{sing}]\!] \vee s \in [\![\mathsf{dance}]\!]\} \qquad \mathsf{Ms}$$

The semantic rule for disjunction also subsumes a Hamblin-style alternative semantics for individual disjunctions, according to which they express an alternative set containing the referents of the disjuncts (Alonso-Ovalle 2006 based on Hamblin 1973; see also Kratzer & Shimoyama 2002; Menéndez-Benito 2005; Charlow 2014, 2020):

(46) **Disjoining names**

$$[\![\mathsf{Apollo\ or\ Ceres}]\!] \;=\; \uparrow [\![\mathsf{Apollo}]\!] \cup \uparrow [\![\mathsf{Ceres}]\!] \;=\; \{\text{Apollo}, \text{Ceres}\} \qquad \mathsf{Me}$$

There is a compositional issue that needs to be addressed. Note that individual disjunctions such as *Apollo or Ceres* cannot be integrated through Functional Application with a verbal projection of functional type $\mathsf{e} \to \mathsf{Ms}$ since this expects an e-type argument instead of an Me-type argument (in fact, the same type mismatch arises with individual conjunctions, which denote singleton sets of plural entities rather than plural entities themselves). To resolve this problem, I assume that composition can proceed via an upgrade to Functional Application which threads nondeterminism through a semantic derivation. In standard architectures for alternative semantics, both functions and their arguments come in alternative sets, and these sets are composed using ‘Pointwise’ Functional Application (Hamblin 1973; Rooth 1985). I designate Pointwise FA using the notation (<*>) for the functional programming concept of an *applicative functor*, as the set type S is an applicative functor precisely because it supports this method along with the type-lifter $\uparrow$ (known as ‘pure’ in applicative programming):

(47) **Pointwise FA**

$$\texttt{<*>} \quad := \quad \lambda F \lambda X.\{f(x) : f \in F, x \in X\} \qquad \texttt{<*>} :: \mathsf{S(a \to b) \to Sa \to Sb}$$

Now, Pointwise FA is not what we need for our purposes. However, S is not only an applicative functor but also what is called a *monad*, as it supports both $\uparrow$ (referred to as ‘return’ in monadic programming) and the following method ($\gg=$), known as ‘Bind’:

(48) **Bind**

$$\gg= \quad := \quad \lambda X \lambda f. \bigcup_{x \in X} f(x) \qquad\qquad (\gg=) :: \mathsf{Sa \to (a \to b) \to Sb}$$

This is what we need to compose entity menus with verbal projections. I assume that, in addition to ordinary Functional Application, composition can proceed by leveraging the monadic character of sets through Bind (to be refined further; see Charlow 2014, 2020 for discussion and additional motivation for using Bind, rather than Pointwise FA, to handle alternatives in natural language processing).

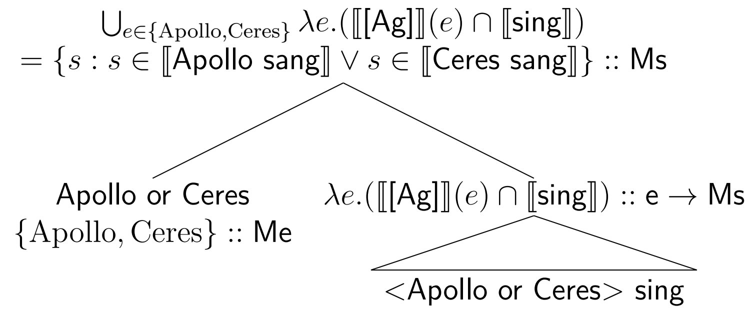

Figure 2: Derivation of Apollo or Ceres sang

In the special case where the initial argument for Bind is a singleton set, this method simplifies to an instance of Functional Application:

(49) $\{x\} \gg= f \quad = \quad f(x)$

Composition with a proper name conjunction via Bind thus resembles Functional Application with a proper name itself:

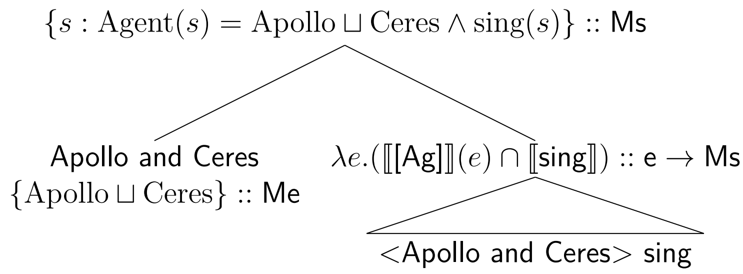

Figure 3: Derivation of Apollo and Ceres sang

Furthermore, the verb *sing* distributes down to atomic individuals on its Agent role, a property that Champollion (2017) refers to as "stratified distributive reference". Consequently, we can infer that any truthmaker for *Apollo and Ceres sang* is composed of a state of Apollo singing together with a state of Ceres singing.

5 Menu semantics for negation

In the standard 'bilateral' sentential truthmaker semantics, an atomic sentence p is directly assigned both a set $|p|^+$ of truthmakers, or exact verifiers, along with a set $|p|^-$ of falsemakers, or exact falsifiers, with a double-entry system of truthmaking $\Vdash$ and falsemaking $\dashv\!|$ clauses projecting these assignments to logically complex sentences (Van Fraassen 1969; Fine 2017a,b,c; Yablo 2014). In this semantics, negation has a straightforward interpretation as a 'flip' operation, toggling between the truthmakers and falsemakers of a sentence: a truthmaker for a negation is a falsemaker for the negated sentence, and vice versa. The heavy lifting is deferred to the falsification clauses for other connectives, where a falsemaker for a conjunction is a falsemaker for a conjunct, and a falsemaker for a disjunction is a fusion of falsemakers for the disjuncts:

$$
\begin{array}{lll}
s \Vdash p & \text{iff} & s \in |p|^+ \\
s \dashv\!| \, p & \text{iff} & s \in |p|^- \\
s \Vdash \neg\varphi & \text{iff} & s \dashv\!| \, \varphi \\
s \dashv\!| \, \neg\varphi & \text{iff} & s \Vdash \varphi \\
s \Vdash \varphi \wedge \psi & \text{iff} & \exists t \exists u (s = t \sqcup u \text{ and } t \Vdash \varphi \text{ and } u \Vdash \psi) \\
s \dashv\!| \, \varphi \wedge \psi & \text{iff} & s \dashv\!| \, \varphi \text{ or } s \dashv\!| \, \psi \\
s \Vdash \varphi \vee \psi & \text{iff} & s \Vdash \varphi \text{ or } s \Vdash \psi \\
s \dashv\!| \, \varphi \vee \psi & \text{iff} & \exists t \exists u (s = t \sqcup u \text{ and } t \dashv\!| \, \varphi \text{ and } u \dashv\!| \, \psi)
\end{array}
$$

Fine (2017a, Section 4) suggests how these recursive clauses may be repurposed at the level of the determiner phrase to determine which entities nominal expressions denote or "anti-denote", with the relation of anti-denotation explicated using negative individuals. We have interpretations such as the following, in which negative individuals play a role analogous to that of falsemakers at the sentential level:

(50)
$$\begin{aligned} [\![\textsf{not Apollo}]\!] &= \{\neg\text{Alfonso}\} \\ [\![\textsf{not(Apollo and Ceres)}]\!] &= \{\neg\text{Apollo}, \neg\text{Ceres}\} \\ [\![\textsf{not(Apollo or Ceres)}]\!] &= \{\neg\text{Apollo} \sqcup \neg\text{Ceres}\} \end{aligned}$$

Inspired by this, one might attempt—as I did in earlier stages of this project—to develop a compositional bilateral version of truthmaker semantics, which determines both a positive and negation interpretation for each expression, ultimately pairing any sentence with a menu of states for achieving truth and a menu of states for achieving falsity.

This bilateral approach, however, faces considerable difficulties. One major challenge arises when we try to compose non-positive entities with complex predicates—a challenge I address later, since it also arises in my own 'unilateral' system. For now, I want to focus on a different issue.

Note the conjunction we get from the generalized cross-categorial bilateral truthmaker semantics is effectively the De Morgan dual of disjunction, where a negated conjunction is equivalent to a disjunction of its negated conjuncts, and a negated disjunction is equivalent to a conjunction of its negated disjuncts. This might be seen as a virtue, since De Morgan-style reasoning works well in many cases. However, I have come to believe, through my work on this project, that the disjunctive treatment of negated conjunctions is really only appropriate in cases where a conjunction is intended to be interpreted distributively. When negated conjunctions give rise to collectivity effects, this treatment can lead to incorrect predictions. Consider the following examples involving the collective predicates *is a married couple* and *sang a duet*:

(51) Not Apollo and Daphne but Ceres and Jupiter are a married couple.

(52) Apollo and Bacchus but not Ceres and Diana sang a duet.

If, as Toosarvandani (2013) argues, the corrective *but* in (51) is a subclausal coordinator, conjoining *not Apollo and Daphne* with *Ceres and Jupiter*, we should avoid interpreting the former negated conjunction in a way that would lead to the absurd disjunctive implication that either Apollo or Daphne, on their own, is not a married couple. Likewise, if, as Vicente (2010) leaves open, the counterexpectational *but* in (52) is subclausal, we do not want to predict the disjunctive implication that Ceres or Diana, on their own, didn't sing a duet.

The conjunction from standard truthmaker semantics now appears as a kind of mongrel conjunction—collective or non-Boolean on its positive side, making it well-suited for capturing collectivity effects in natural language, but distributive or Boolean on its negative side, which does not align well with negated conjunctions in collective contexts. This difficulty led me to abandon the bilateral approach altogether and instead pursue a different theory, one that interprets negation unilaterally with a more restricted commitment to De Morgan.

While the semantic rules for *and* and *or* in Section 4 were relatively straightforward, the rule for *not* is more complex. Therefore, I would like to gradually develop the intuition behind my treatment of negation in stages by considering increasingly general cases.

Stage I. To begin, consider an expression α whose denotation is, or can be lifted to, a singleton set $\{x\}$ where x is either a positive or negative entity, or a positive or negative state (i.e., x is not a fusion of negative elements nor a hybrid element). What does *not* α signify? In this base case, my answer realizes the traditional idea that the function of negation is to flip polarity. I propose that *not* α signifies $\{\neg x\}$. Notably, negated names denote negative individuals, as in (50), however negative individual conjunctions now denote negative plural entities, differing from (50), which followed De Morgan reasoning:

(53)
$$\begin{aligned} [\![\textsf{not Apollo}]\!] &= \{\neg\text{Apollo}\} \\ [\![\textsf{not(Apollo and Ceres)}]\!] &= \{\neg(\text{Apollo} \sqcup \text{Ceres})\} \end{aligned}$$

Stage II. Next, suppose that the denotation, or lifted denotation, of α is a set $\{x, y, ...\}$, which may or may not be a singleton, and consists of positive or negative elements x, y, ... that can vary in polarity (i.e., still no fusions of negative elements nor hybrids). This includes the previous case and extends to expressions such as individual disjunctions that do not denote singletons. In this more general case, I propose that *not* α signifies the singleton set $\{\neg x \sqcup \neg y \sqcup ...\}$, which contains as its sole member the fusion of the orthogonal counterparts of the elements in α's denotation. The idea behind this follows the De Morgan principle stating that the negation of a disjunction is equivalent to the conjunction of the negated disjuncts—namely, that negating an expression that offers a set of alternative options removes all of these options from the menu. While I previously took issue with the equivalence of a negated conjunction with the disjunction of the negated conjuncts, I think the other De Morgan principle invoked here is unproblematic, since disjunction in natural language is a distributive item, unlike conjunction.

(54) $$\begin{aligned} [\![\textsf{Apollo or Ceres}]\!] &= \{\text{Apollo}, \text{Ceres}\} \\ [\![\textsf{not(Apollo or Ceres)}]\!] &= \{\neg\text{Apollo} \sqcup \neg\text{Ceres}\} \end{aligned}$$

For current purposes, this second layer of generality is all we need. None of the examples to follow in this paper extend beyond it. However, for thoroughness, I will also consider the most general case.

Stage III. What if the elements x, y, ... are not exclusively positive or negative entities or states, but can also be fusions of negative elements, such as $\neg\text{Apollo} \sqcup \neg\text{Bacchus}$, or hybrid elements that combine positive and negative parts, such as $\text{Bacchus} \sqcup \neg\text{Ceres}$? Interestingly, while individual conjunctions can behave collectively, expressions that I take to denote fusions of negative entities or states, like *neither Apollo nor Bacchus*, as well as those denoting hybrid entities or states, like *Bacchus but not Ceres*, are distributive items. For these cases, then, I think we can apply the De Morgan principle equating the negation of a conjunction with the disjunction of the negated conjuncts. To implement this, I assign each element x, y, ... in α's denotation an 'orthogonality set' which contains the orthogonal counterpart of each of its non-null maximal positive or negative parts—where a maximal positive or negative part of an entity or state is one that does not contain another positive or negative element as a proper part. For a non-null positive or negative element x, the orthogonality set is simply $\{\neg x\}$. However, a fusion of negative elements or a mixed polarity element may have more than one member in its orthogonality set. For instance, the orthogonality set for $\neg\text{Apollo} \sqcup \neg\text{Bacchus}$ is $\{\text{Apollo}, \text{Bacchus}\}$, while that for $\text{Bacchus} \sqcup \neg\text{Ceres}$ is $\{\neg\text{Bacchus}, \text{Ceres}\}$. Then, reapplying the uncontroversial De Morgan reasoning from Stage II, I take *not* α to signify the menu consisting of all ways of choosing an item from each of the orthogonality sets for x, y, ... and fusing these selections together.

In formal terms, I understand the use of *not* in English as a function that maps a set of entities or states to another set of entities or states, where each element in the output set is a sum of elements selected from each of the orthogonality sets for members of the function's argument. To implement this proposal, we need to establish some terminology. Let $\text{PART}^+(x)$ denote the set of non-null maximal positive parts of an entity or state x. This set includes y when $y \leqslant x$, y is a non-null positive element, and there is no z such that $z \leqslant x$, z is a positive element, and $y < z$. Since the set of positive entities or states forms a complete join semilattice, $\text{PART}^+(x)$ is either a singleton or empty. Similarly, let $\mathrm{PART}^{-}(x)$ denote the set of non-null maximal negative parts of x, which may have more than one member. Finally, let $\mathrm{PART}(x)$ be the union $\mathrm{PART}^{+}(x) \cup \mathrm{PART}^{-}(x)$.

We can formally define the auxiliary notion of an orthogonality set as follows:

(55) The **orthogonality set** for an entity or state x is
$$\mathcal{O}(x) \quad := \quad \{\neg y : y \in \mathrm{PART}(x)\}$$

My proposal is that negation applies the pointwise fusion operation to the family of orthogonality sets determined by the entities or states in its menu type argument:

(56) **Generalized negation**
$$[\![\textsf{not}]\!] \quad = \quad \lambda X. \dot{\bigsqcup}_{x \in X} \mathcal{O}(x) \qquad \textsf{Ma} \to \textsf{Ma}$$

Recycling terminology from Bernard & Champollion (2024), I abbreviate this lambda function as NEG.

Here are some examples. When an atomic sentence whose truthmaker content consists of a set of positive states is negated, the negation expresses a singular menu whose sole option is the fusion of the negative orthogonal counterparts of these positive states:

(57) **Negating sentences**
$$\begin{aligned} [\![\textsf{not(Apollo sang)}]\!] &= \mathrm{NEG}([\![\textsf{Apollo sang}]\!]) \\ &= \{\textstyle\bigsqcup\{\neg s : s \in [\![\textsf{Apollo sang}]\!]\}\} \qquad \textsf{Ms} \end{aligned}$$

Likewise, when a verb like *sing* is negated, the negation expresses a singleton set containing the fused orthogonal counterparts of all the singing events:

(58) **Negating verbs**
$$\begin{aligned} [\![\textsf{not sing}]\!] &= \mathrm{NEG}([\![\textsf{sing}]\!]) \\ &= \{\textstyle\bigsqcup\{\neg s : s \in [\![\textsf{sing}]\!]\}\} \qquad \textsf{Ms} \end{aligned}$$

The semantic rule for negation also extends to negated names. As was the case with individual conjunctions and disjunctions, the referent of a name must first be lifted with Set Return before NEG can be applied:

(59) **Negating names**
$$\begin{aligned} [\![\textsf{not Apollo}]\!] &= \mathrm{NEG}(\uparrow [\![\textsf{Apollo}]\!]) \\ &= \{\neg \mathrm{Apollo}\} \qquad \textsf{Me} \end{aligned}$$

I suspect this allowance for negated names will face resistance give that, in English, the word *not* cannot usually modify a proper name occurring on its own, whether it is in the subject or object position:

(60) a. *Not Napoleon won at Waterloo.
b. *Joséphine loves not Napoleon.

Admittedly, I am not sure how to explain this distributional restriction on negation. However, I think it poses a puzzle for everyone. The classical theory that deals with, for example, conjunctions of names with quantifiers has to allow for Montague's type-shifting of entities onto generalized quantifiers. Therefore, classical theorists also face the challenge of explaining why generalized Boolean negation cannot apply to a lifted entity in examples like (60).[11]

In any case, the constituent negation of proper names *does* arguably occur in certain environments, such as the adversative constructions (27-a) and (28-a) discussed in Section 2, repeated below in (61) (Vicente 2010; Toosarvandani 2013):

(61) a. Lysander loves not Hermia but Helena.
b. Hermia but not Helena is coming to the wedding.

Entity negation will enable me to analyze these examples as instances of collective subclausal conjunction, as illustrated by:

(62)
$$\begin{aligned} [\![\mathsf{not(Hermia)\ but\ Helena}]\!] &= \mathrm{NEG}(\uparrow [\![\mathsf{Hermia}]\!]) \sqcup \uparrow [\![\mathsf{Helena}]\!] \\ &= \{\neg\mathrm{Hermia} \sqcup \mathrm{Helena}\} \end{aligned}$$
Me

In the following section, I will additionally provide a new analysis of negative quantifiers such as *no soldier* in terms of entity negation:

(63) No soldier escaped the Battle of Waterloo unscathed.

By doing so, I will be able to present a collective treatment of coordinations in which these negative quantifiers occur.

Before proceeding further, I would like to address a few of the remaining empirical and theoretical concerns with my use of negative entities. First, some correspondents have expressed a worry regarding anaphora. If negated names denote negative individuals, as I propose, one might expect these negative individuals to be available for subsequent anaphora, similar to the positive referents of ordinary proper

[11] I am grateful to Patrick Elliot for discussing this point with me. Collins (2016) argues that examples like (60) demonstrate that proper names cannot generally be shifted into the type of generalized quantifiers; however, it is then unclear how he intends to interpret conjunctions of names with quantifiers.

names. Negative entities, however, seem excluded from the possible interpretations of anaphoric pronouns:

(64) Hermia but not Helena is coming. They were invited.

While (64) allows for plural anaphora, it only has a reading where both Hermia and Helena were invited, as predicted when *they* resolves to the positive entity Hermia $\sqcup$ Helena. Crucially, (64) does not support a reading where Hermia was invited but Helena was not, which would be expected if *they* could also retrieve the hybrid entity Hermia $\sqcup \neg$Helena.

This concern about anaphora presupposes that the positive, negative, and hybrid entities in type $\mathtt{e}$ should not differ significantly in their anaphoric potential—a presupposition that seems reasonable if they are ontologically on a par. However, as discussed in Section 3, I do not consider negative entities as part of natural language metaphysics; rather, I view the polarization of the entity domain as a means to model the empirical assumption that English speakers track negation through polarity marking on entities and states in semantic processing. From this perspective, examples like (64) simply reveal something about this proposed system of polarity marking: while polarity marking can factor into the compositional determination of meaning, negative marking does not survive under anaphora. Negative entities might be likened to a negative balance in a bank account, which is crucial for calculating the total wealth of an account holder, even though certain activities possible with a positive bank balance, such as withdrawing and spending, are not possible with a negative balance.[12] (For those still worried about the absence of negative anaphora, I would also add that, as suggested in Section 3, I think we could reformulate the theory presented in this article by working with standard domains containing only positive elements and allowing the compositional semantics itself to introduce and operate on ordered pairs like $\langle \text{Apollo}, 1\rangle$ and $\langle \text{Ceres}, 0\rangle$.)

Another concern I have come across with negative individuals pertains to number agreement. If *not Helena* in (64) denotes an individual, albeit a negative one, that contributes to a plurality, albeit a hybrid one, one might expect the verb in this sentence to permit plural rather than singular number marking. Yet, this is not the case:

(65) *Hermia but not Helena are coming.

The ungrammaticality of plural marking in (65) may, however, be explained by the fact that adversative coordinations like *Hermia but*

[12] Thanks to Kit Fine for suggesting this analogy.

not Helena function as distributive items, similar to *neither…nor* constructions (e.g., *neither Hermia nor Helena*), which also require singular marking. I suspect the same interaction between distributivity and number agreement that mandates singular marking with *neither…nor* also necessitates it with adversatives.

There are also some technical compositional difficulties that need to be addressed. As pointed out by Bernard & Champollion (2018, 2024), the flat-footed use of NEG for VP negation can lead to erroneous results. Although Bernard & Champollion's NEG function differs from mine, the problem arises in my framework:

(66) Ceres didn't sing.

(67)
$$\begin{aligned} & [\![[[\mathsf{Ag}][_{\mathrm{DP}}\mathsf{Ceres}]][_{\mathrm{NegP}}\mathsf{not\ sing}]]]\!] \\ = \; & [\![\mathsf{Ag}]\!]([\![\mathsf{Ceres}]\!]) \cap \mathrm{NEG}([\![\mathsf{sing}]\!]) \\ = \; & \{s : \mathrm{Agent}(s) = \mathrm{Ceres} \wedge s = \bigsqcup\{\neg s' : s' \in [\![\mathsf{sing}]\!]\}\} \quad \mathsf{Ms} \end{aligned}$$

This is not what we want. Sentence (66) is about Ceres, however the truthmaking condition in (67) includes the fusion of the orthogonal counterparts of *all* singings, whether or not they involve Ceres (and as a result, the set of computed truthmakers is presumably empty). What we want instead is for the semantic scope of NEG to include the subject. In their earlier work, Bernard & Champollion (2018) propose a directly compositional semantic solution to this scope mismatch by allowing negation to signify a higher-order function that takes a verb phrase and a subject as its argument, combines them internally, and then applies NEG to the outcome of this combination. In more recent work, Bernard & Champollion (2024) resolve the scope issue by appealing to the VP-internal subject hypothesis of Koopman & Sportiche (1991), which I also adopt in this paper. According to this syntactic strategy, subjects move out of a verbal projection and past negation, allowing negation to scope above the subject as required:

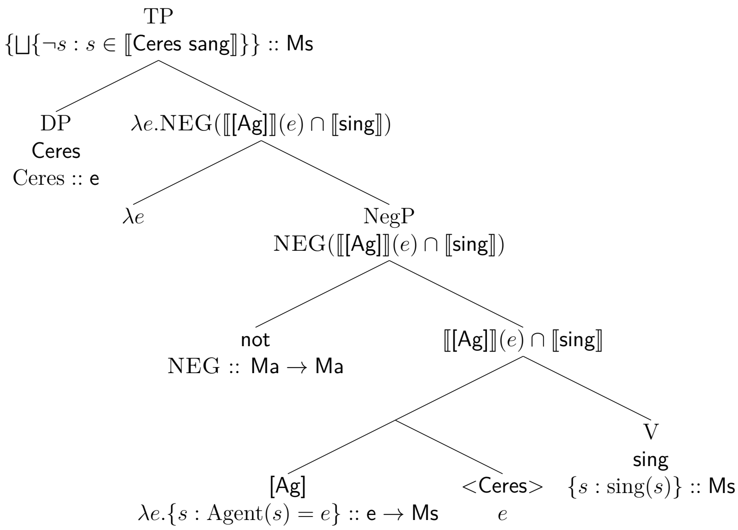

Figure 4: Derivation of Ceres didn't sing

There are other compositional difficulties that arise when NEG introduces negative entities, but these require a more complex solution. I will therefore defer discussion until Section 7, after introducing a menu semantics for certain 'quantificational' expressions, including *no*.

6 Menu semantics for quantifiers

Fine (2017c) proposes a reduction of the semantics of universally and existentially quantified statements to the semantics of sentential conjunctions and disjunctions, respectively. For now, let us assume that the set of positive atomic individuals in the entity domain, which I denote as IN^+, serves as a fixed domain of quantification, and we have constants e_1, e_2, ... which refer to all the individuals e_1, e_2, ... that belong to IN^+. Fine presents both truthmaking $\Vdash$ and falsemaking $\dashv\mid$ clauses for quantifiers, but I only present his truthmaking clauses:

(68)
$$\begin{array}{lll} s \Vdash \forall x \varphi(x) & \text{iff} & s \Vdash \varphi(\mathsf{e}_1) \wedge \varphi(\mathsf{e}_2) \wedge ... \\ & \text{iff} & \exists s_1 \exists s_2 ...(s_1 \Vdash \varphi(\mathsf{e}_1), s_2 \Vdash \varphi(\mathsf{e}_2), ... \wedge s = s_1 \sqcup s_2 \sqcup ...) \\ s \Vdash \exists x \varphi(x) & \text{iff} & s \Vdash \varphi(\mathsf{e}_1) \vee \varphi(\mathsf{e}_2) \vee ... \\ & \text{iff} & s \Vdash \varphi(\mathsf{e}_i) \text{ for some } e_i \in \text{IN}^+ \end{array}$$

In this paper, I assume that Fine's truthmaking conditions are essentially correct, and I adopt his overall approach of using the semantics of conjunction and disjunction to inform the analysis of quantified determiner phrases (such as *some* and *every* phrases, as well as other determiner phrases like definite and indefinite descriptions). However, unlike Fine, I aim to derive the truthmaking conditions in (68) in a fully compositional manner. To this purpose, my treatment of quantificational expressions in English is based most directly, not on the semantics of *sentential* conjunctions and disjunctions, which signify menus of states, but rather on my semantics for *individual* conjunctions and disjunctions, which signify menus of entities.

The semantics for quantifiers presented in this section is non-Montagovian. While Montague (1973), as well as Barwise & Cooper (1981), Keenan & Faltz (1985), Keenan & Stavi (1986), and many other semanticists, assimilated proper names to quantifiers by assigning names the higher-order function type $(\mathsf{e} \to \mathsf{t}) \to \mathsf{t}$ of generalized quantifiers (or "property sets"), I do things the other way round by assigning entity-style denotations to 'quantificational' DPs. Here is my initial formulation of a menu semantics for *every thing* and *some thing* (subject to further refinement):

(69) **Unrestricted universal and existential quantifiers**

$$\begin{array}{lcll} [\![\textsf{every thing}]\!] & = & \{\bigsqcup \mathrm{IN}^+\} & \\ & = & \{\text{Björk} \sqcup \text{Koh-i-Noor} \sqcup \text{Venus} \sqcup ...\} & \textsf{Me} \\ [\![\textsf{some thing}]\!] & = & \mathrm{IN}^+ & \\ & = & \{\text{Björk}, \text{Koh-i-Noor}, \text{Venus}, ...\} & \textsf{Me} \end{array}$$

On this preliminary proposal, universally quantified DPs generalize individual conjunctions, while existentially quantified DPs generalize individual disjunctions.

With the entry for *some thing* in (69), we can recover Fine's truthmaking condition for existential quantification in (68) in a compositional manner:

(70) a. Some thing vanished.

b.
$$\begin{array}{cll} & [\![[[_{\mathrm{DP}}\textsf{Some thing}][\lambda e[_{\mathrm{TP}}[[\textsf{Th}]e][_{\mathrm{V}}\textsf{vanished}]]]]]\!] & \\ = & [\![\textsf{some thing}]\!] \gg= \lambda e.([\![[\textsf{Th}]]\!](e) \cap [\![\textsf{vanish}]\!]) & \\ = & \bigcup_{e \in \{\text{Björk},\text{Koh-i-Noor},...\}}\{s : \text{Theme}(s) = e \wedge \text{vanish}(s)\} & \\ = & \{s : \text{Theme}(s) = \text{Björk} \wedge \text{vanish}(s)\} \cup & \\ & \{s : \text{Theme}(s) = \text{Koh-i-Noor} \wedge \text{vanish}(s)\} \cup ... & \\ = & \{s : s \in [\![\textsf{Björk vanished}]\!] \vee & \\ & s \in [\![\textsf{Koh-i-Noor vanished}]\!] \vee ...\} & \textsf{Ms} \end{array}$$

However, the current proposal for *every thing* cannot be quite right. Unlike individual conjunctions, *every thing* is a distributive item. When this universal quantifier phrase combines with a predicate, the predicate is generally understood to hold separately of each individual in the quantificational domain. The entry provided in (69) fails to capture this distributivity and instead resembles the classic collective theory of definite descriptions (Sharvy 1980; Link 1983), which pattern with individual conjunctions in being compatible with collective predicates:

(71) a. Houdini and Presto {vanished/gathered/are a duo}.
b. The magicians {vanished/gathered/are numerous}.
c. Every thing {vanished/*gathered/*are numerous}.

Now, I am not looking to get into the weeds about the proper treatment of distributivity, which is largely peripheral to the story I am trying to tell about negative entities. To derive the appropriate truthmakers for simple universally quantified statements, I will simply make the assumption that English *every* requires its nuclear scope to be headed by a distributivity operator (Beghelli & Stowell 1997; Brasoveanu 2013; Champollion 2017, Section 9.5; among many others). We can employ a variant of Link's (1987) atomic D operator for this purpose, where $\mathrm{AT}(x)$ is the set of atomic parts of x:

(72) **Distributivity operator**

$$\mathrm{D} \;:=\; \lambda f \lambda e. \dot{\bigsqcup}_{e' \in \mathrm{AT}(e)} f(e') \qquad \mathrm{D} :: (\mathsf{e} \to \mathsf{Ss}) \to (\mathsf{e} \to \mathsf{Ss})$$

(73) a. Every thing vanished.
b.
$$\begin{aligned}
& [\![[[_{\mathrm{DP}}\textsf{Every thing}][\mathrm{D}[\lambda e[_{\mathrm{TP}}[[\textsf{Th}]e][_{\mathrm{V}}\textsf{vanished}]]]]]]]\!] \\
= \;& [\![\textsf{every thing}]\!] \gg\!= \lambda e. \dot{\bigsqcup}_{e' \in \mathrm{AT}(e)} ([\![[\textsf{Th}]]\!](e') \cap [\![\textsf{vanish}]\!]) \\
= \;& \dot{\bigsqcup}_{e \in \{\text{Björk}, \text{Koh-i-Noor}, \ldots\}} \{s : \mathrm{Theme}(s) = e \wedge \mathrm{vanish}(s)\} \\
= \;& \{s : \mathrm{Theme}(s) = \text{Björk} \wedge \mathrm{vanish}(s)\} \dot{\sqcup} \\
& \{s : \mathrm{Theme}(s) = \text{Koh-i-Noor} \wedge \mathrm{vanish}(s)\} \dot{\sqcup} \ldots \\
= \;& \{s_1 \sqcup s_2 \sqcup \ldots : s_1 \in [\![\textsf{Björk vanished}]\!], \\
& s_2 \in [\![\textsf{Koh-i-Noor vanished}]\!], \ldots\} \qquad \mathsf{Ms}
\end{aligned}$$

The presence of D inside a nuclear scope also ensures that *every thing* cannot combine with collective predicates, which require states with semantically plural participants.

We now have the beginnings of a more general theory of denoting DPs. However, the theory is oversimplified in at least two significant respects. First, it assumes a fixed quantificational domain. Second, it only considers vacuous restriction. To allow for variable domains and non-vacuous restriction, Fine (2017c) introduces *totality facts*. For each formula $\varphi(x)$ and subdomain of individuals $E \subseteq \mathrm{IN}^+$, the totality fact $\tau_{|\varphi(x)|,E}$ represents that the members e_1, e_2, ... of E are exactly the individuals that satisfy φ. On one way of implementing Fine's approach, such totality facts serve as enabling preconditions for other states to serve as truthmakers for quantified sentences (see Yablo 2014, Section 4.4 for essentially the same proposal):

(74) $\tau, s \Vdash \varphi \quad := \quad s$ verifies φ conditional on τ obtaining

(75) $\tau, s \Vdash \forall x(\varphi(x) : \psi(x))$ iff $\tau = \tau_{|\varphi(x)|,E}$ and $\exists s_1 \exists s_2 ...(s_1 \Vdash \psi(\mathsf{e}_1), s_2 \Vdash \psi(\mathsf{e}_2), ... \wedge s = s_1 \sqcup s_2 \sqcup ...)$

I take a different, though related, approach that doesn't require bringing in totality facts as special devices in the model theory. Drawing on von Fintel (1994) and much subsequent research in linguistics, I posit that quantifiers come with a hidden state argument s^* supplied by context or preceding linguistic material. This provides all the variability we need. The following entries are for *every* and *some* in combination with a morphologically singular noun phrase α_{sing}:

(76) **Restricted universal and existential quantifiers**

$$[\![\mathsf{every}_{s^*}(\alpha_{\text{sing}})]\!] = \{\bigsqcup\{e \in \text{IN}^+ : \exists s \leqslant s^*(\text{Participant}(s) = e \wedge s \in [\![\alpha]\!])\}\}$$
$$[\![\mathsf{some}_{s^*}(\alpha_{\text{sing}})]\!] = \{e \in \text{IN}^+ : \exists s \leqslant s^*(\text{Participant}(s) = e \wedge s \in [\![\alpha]\!])\}$$

(77)
$$\begin{aligned} & [\![\mathsf{every}_{s^*}(\mathsf{magician})]\!] \\ = \ & \{\bigsqcup\{e \in \text{IN}^+ : \exists s \leqslant s^*(\text{Participant}(s) = e \wedge s \in [\![\mathsf{magician}]\!])\}\} \\ = \ & \{\text{Houdini} \sqcup \text{Presto} \sqcup ...\} \qquad \mathsf{Me} \end{aligned}$$

(78)
$$\begin{aligned} & [\![\mathsf{some}_{s^*}(\mathsf{magician})]\!] \\ = \ & \{e \in \text{IN}^+ : \exists s \leqslant s^*(\text{Participant}(s) = e \wedge s \in [\![\mathsf{magician}]\!])\} \\ = \ & \{\text{Houdini}, \text{Presto}, ...\} \qquad \mathsf{Me} \end{aligned}$$

As before, it is assumed that *every* DPs combine with distributivized nuclear scopes, while *some* DPs do not:

(79) a. Every magician vanished.

b.
$$\begin{aligned} & [\![[[_{\text{DP}}\mathsf{Every}_{s^*}\ \mathsf{magician}][\text{D}[\lambda e[_{\text{TP}}[[\mathsf{Th}]e][_{\text{V}}\mathsf{vanished}]]]]]]\!] \\ = \ & \dot{\bigsqcup}_{e \in \{\text{Houdini},\text{Presto},...\}}([\![[\mathsf{Th}]]\!](e) \cap [\![\mathsf{vanish}]\!]) \\ = \ & \{s_1 \sqcup s_2 \sqcup ... : s_1 \in [\![\mathsf{Houdini\ vanished}]\!], \\ & s_2 \in [\![\mathsf{Presto\ vanished}]\!], ...\} \qquad \mathsf{Ms} \end{aligned}$$

(80) a. Some magician vanished.

b.
$$\begin{aligned} & [\![[[_{\text{DP}}\mathsf{Some}_{s^*}\ \mathsf{magician}][\lambda e[_{\text{TP}}[[\mathsf{Th}]e][_{\text{V}}\mathsf{vanished}]]]]]\!] \\ = \ & \bigcup_{e \in \{\text{Houdini},\text{Presto},...\}}([\![[\mathsf{Th}]]\!](e) \cap [\![\mathsf{vanish}]\!]) \\ = \ & \{s : s \in [\![\mathsf{Houdini\ vanished}]\!] \vee \\ & s \in [\![\mathsf{Presto\ vanished}]\!] \vee ...\} \qquad \mathsf{Ms} \end{aligned}$$

Since the determiner *every* is grammatically singular and does not permit a plural restrictor, the entry in (76) is sufficient. On the other hand, *some* is capable of combining with plural noun phrases, as with *some things* or *some magicians*. The Me-type treatment in (76) extends naturally to cover these cases: $[\![\mathsf{some}_{s^*}(\alpha)]\!]$ is $[\![\mathsf{some}_{s^*}(\alpha_{\mathrm{sing}})]\!]$ but without the requirement that elements in the alternative set satisfying the restriction be single individuals.

We come finally to the case of negative quantificational DPs such as *no thing* and *no magician*. Here negative entities re-enter the picture. As mentioned in Section 2, a number of linguists have proposed that *no* and other "n-words" can be analyzed as indefinites occurring within the scope of a sentential negation operator (Klima 1964; Ladusaw 1992; Zeijlstra 2004; Landman 2004; Penka 2011; Brasoveanu et al. 2013). In my system, this scope-splitting analysis yields satisfactory results:

(81) a. No magician vanished.

b. $$\begin{aligned} & [\![[_{\mathrm{NegP}}\mathsf{not}[[_{\mathrm{DP}}\mathsf{some}_{s_0}(\mathsf{magician})][\lambda e[_{\mathrm{TP}}[[\mathsf{Th}]e][_{\mathrm{V}}\mathsf{vanished}]]]]]]\!] \\ = \ & \mathrm{NEG}(\{s : s \in [\![\mathsf{Houdini\ vanished}]\!] \vee \\ & s \in [\![\mathsf{Presto\ vanished}]\!] \vee ...\}) \\ = \ & \{\bigsqcup\{\neg s : s \in [\![\mathsf{Houdini\ vanished}]\!] \vee \\ & s \in [\![\mathsf{Presto\ vanished}]\!] \vee ...\}\} \qquad \mathsf{Ms} \end{aligned}$$

However, I would like to pursue an alternative scope-splitting analysis based on constituent rather than sentential negation. In this analysis, *no thing* is understood to have the underlying Me-type form *not*(*some thing*), and similarly for non-vacuously restricted negative quantifiers:

(82) **Negative quantifiers**

$$\begin{aligned} [\![\mathsf{no}_{s^*}(\alpha_{\mathrm{sing}})]\!] \ &= \ [\![[_{\mathrm{NegP}}\mathsf{not}[_{\mathrm{DP}}\mathsf{some}_{s^*}(\alpha_{\mathrm{sing}})]]]\!] \\ &= \ \mathrm{NEG}(\{e \in \mathrm{IN}^+ : \exists s \leqslant s^*(\mathrm{Participant}(s) = e \wedge s \in [\![\alpha]\!])\}) \\ &= \ \{\bigsqcup\{\neg e : e \in \mathrm{IN}^+ \wedge \exists s \leqslant s^*(\mathrm{Participant}(s) = e \wedge s \in [\![\alpha]\!]\}\} \quad \mathsf{Me} \end{aligned}$$

(83) $$\begin{aligned} [\![\mathsf{no}_{s^*}(\mathsf{magician})]\!] \ &= \ [\![[_{\mathrm{NegP}}\mathsf{not}[_{\mathrm{DP}}\mathsf{some}_{s^*}(\mathsf{magician})]]]\!] \\ &= \ \{\neg\mathrm{Houdini} \sqcup \neg\mathrm{Presto} \sqcup ...\} \qquad \mathsf{Me} \end{aligned}$$

The remaining challenge is to demonstrate how such entity menus populated by negative individuals can effectively combine with the state menus contributed by verbal and other predicative projections in order to provide suitable truthmakers for negatively quantified statements. This task is not as straightforward as it may initially appear.

7 Composing negative entities

Peter Geach, in *Reference and Generality*, Section 27 (1962), presents the following argument supporting the Aristotelian thesis that we can negate the predicate but not the subject of a predication (see Geach

1972, Section 1.5, for a historical argument tracing this thesis back to Aristotle's *De Interpretatione*):[13]

> When a proposition is negated, the negation may be taken as going with the predicate in a way in which it cannot be taken to go with the subject. For predicables always occur in contradictory pairs; and by attaching such a pair to a common subject we get a contradictory pair of propositions. But we never have a pair of names so related that by attaching the same predicates to both we always get a pair of contradictory propositions.
>
> It is easy to prove this formally. The conjunction of a pair of predicables when attached to a name "x" signifies the same as the conjunction of the propositions that we get by attaching each predicable separately to "x"; this is precisely what conjunction means when applied to predicables rather than propositions. Now suppose we had a pair of names "x" and "y" such that by attaching the same predicate to both we always got a pair of contradictory propositions. Thus we have:
>
> "$(P\&Q)x$" is contradictory to "$(P\&Q)y$".
>
> Hence, in view of what the conjunction of predicables, "$P\&Q$", has to mean:
>
> "$Px\&Qx$" is contradictory to "$Py\&Qy$".
>
> But, by our supposition about "x" and "y", "Px" and "Py" are contradictories, and so are "Qx" and "Qy". We may thus infer:
>
> "$Px\&Qx$" is contradictory to "not$(Px)\&$not(Qx)".
>
> And from this it is easily proved, by way of the truth-functional tautology:
>
> $(\sim (p\&q) \equiv (\sim p\& \sim q)) \equiv (p \equiv q)$
>
> that for this name "x" arbitrary predications "Px" and "Qx", assuming they can be significantly formed into one predication, must always have the same truth-value.

[13]Geach presents a different argument in the 1980 emended edition of his book. However, I believe that Geach's 1980 argument can be resisted in a similar manner to how I intend to resist his 1962 argument.

Nowadays, this type of argument might be referred to as a "triviality argument" (cf. Lewis 1976, 1986). Geach's argument against the possibility of negated names, or as support for the claim that names never come in contradictory pairs, first assumes the existence of such a pair of names, denoted as "x" and "y", and then proceeds not towards outright contradiction but rather to the conclusion that the assumption of "x" and "y" is viable only in the trivial situation where, for any predicates "P" and "Q" that can be conjoined into the compound predicate "$P\&Q$", the truth values of "Px" and "Qx" must be the same (and "Py" and "Qy" must also share the same truth value).

A similar argument against negated names can be found in Peter Strawson's *Subject and Predicate in Logic and Grammar* (1974) (with Geach mentioned in a footnote as a source of inspiration):

> As for negative names, suppose we have the following conjunctive sentence:
>
> (1) Fa and Ga
>
> Then, by double negation, this is equivalent to
>
> (2) $\sim (\sim (Fa \text{ and } Ga))$
>
> which, by the introduction of a conjunctive predicate, is equivalent to
>
> (3) $\sim (\sim ((F \text{ and } G)a))$
>
> If we can frame negative subjects, (3) is equivalent to
>
> (4) $\sim ((F \text{ and } G) \sim a)$
>
> and (4) can be expanded into
>
> (5) $\sim (F \sim a \text{ and } G \sim a)$
>
> which will be equivalent to
>
> (6) $\sim (\sim (Fa) \text{ and } \sim (Ga))$
>
> Now (6) is equivalent to
>
> (7) Fa or Ga
>
> But evidently (1) is not equivalent to (7). (p. 6)

Like Geach, we can interpret Strawson as providing a triviality result: if we allow for the possibility of negating the subject term "a", then for any predicates "F" and "G" that can be conjoined, the sentential conjunction "Fa and Ga" and the sentential disjunction "Fa or Ga" are equivalent, however this equivalence holds only for the special case where "Fa" and "Ga" have the same truth value.

Geach's and Strawson's arguments are supposed to show that a language cannot coherently admit negative subject terms, assuming it also allows for conjoined compound predicates. I do not think their arguments establish this limitation on negation. Nevertheless, I do think these arguments raise an important difficulty for compositionality within a semantic theory, such as my own, that allows for negated determiner phrases featuring negative entities in their denotations.

Both arguments rely on the following principle, which Geach suggests is an analytic truth that embodies "what conjunction means when applied to predicables rather than propositions":

(84) **Conjunctive Predication:** Given a name "a" and predicates "P" and "Q" that can attach to this name both individually and as a single predicate conjunction, "$(P\&Q)a$" is logically equivalent to "$Pa\&Qa$".

I agree that when "a" is an ordinary proper name, Conjunctive Predication appears unobjectionable. For instance, asserting that Socrates possesses the compound property of being wise and snub-nosed is essentially equivalent to separately asserting his possession of the property of being wise and the property of being snub-nosed, and then combining the results of these individual predications. However, Geach and Strawson invoke Conjunctive Predication for the more problematic case where "a" is a negative name. In Geach's argument, this principle underlies the inference from the contradictoriness of "$(P\&Q)x$" and "$(P\&Q)y$" to the contradictoriness of "$Px\&Qx$" and "$Py\&Qy$", where the names "x" and "y" are complementary. In Strawson's argument, the principle is used in the transition from (4) to (5).

This is where the arguments presented by Geach and Strawson should be resisted, or so I believe. It is important to note that Conjunctive Predication apparently requires that the negation contributed by a negative name must take scope below the conjunction contributed by a predicative conjunction. However, natural language appears to defy this scopal order. For instance, if Socrates but not Plato is wise and snub-nosed, we can only infer that Plato isn't both wise and snub-nosed, but we cannot conclude that he isn't wise and isn't snub-nosed.[14] Actually, Geach and Strawson are somewhat inconsistent in their treatment of scope. Geach implicitly assumes that the negation from a name can take scope over a conjunctive predicate when he argues

[14]I use a counterexpectational *but* construction because negated names are ungrammatical as isolated subjects. Admittedly, this example poses a challenge to Conjunctive Predication only if its proper analysis involves constituent DP negation.

that "$(P\&Q)x$" and "$(P\&Q)y$" are contradictory, and Strawson assumes this when equating (3) and (4). Small wonder that their use of negative or contradictory names leads to peculiar and problematic results.

A similar difficulty arises for the corresponding principle concerning disjunction:

(85) **Disjunctive Predication:** Given a name "a" and predicates "P" and "Q" that can attach to this name both individually and as a single predicate disjunction, "(P or Q)a" is logically equivalent to "Pa or Qa".

As in the case of conjunction, if "a" is a negative name, invoking this principle leads to problematic implications from the perspective of natural language. It apparently requires that the negation contributed by a negative name must scope below the disjunction contributed by a predicative disjunction. However, if Socrates but not Plato is wise or snub-nosed, we can infer not only that Plato isn't wise or snub-nosed, but also the stronger claim that Plato isn't wise and isn't snub-nosed. This stronger claim is what we get if negation scopes over the disjunction.

The general moral I want to draw from my discussion of Geach's and Strawson's triviality arguments is that when the logical subject of a predication can introduce negation, it is crucial to ensure that this negation scopes above the predicate rather than below it. This is necessary to avoid odd outcomes at best and potential incoherence at worst. In my menu semantics, the problem arises when we attempt to compose the negative entities contributed by negative DPs using the Bind method, as this fails to generate the desired scope-taking behavior.

To illustrate this in a perspicuous way, let me use an example involving the negative quantifier *no magician*, which I previously analyzed in (83) as denoting the singleton set containing the sum of all the anti-magicians. Specially, consider what happens when this quantifier is combined with a predicate disjunction:

(86) a. No magician vanished or turned into a dove.

b. $[\![[[_{\text{NegP}}\textsf{not}[_{\text{DP}}\textsf{some}_{s_0}(\textsf{magician})]]$
$[\lambda e[_{\text{TP}}[[\textsf{Th}]e][_{\text{VP}}\textsf{vanished or turned into a dove}]]]]]\!]$
$= [\![\textsf{not}(\textsf{some}_{s_0}(\textsf{magician}))]\!] \gg=$
$\lambda e.([\![[\textsf{Th}]]\!](e) \cap ([\![\textsf{vanish}]\!] \cup [\![\textsf{turn...}]\!]))$
$= \{\neg\text{Houdini} \sqcup \neg\text{Presto} \sqcup ...\} \gg=$
$\lambda e.\{s : \text{Theme}(s) = e \wedge (\text{vanish}(s) \vee \text{turn-into-a-dove}(s))\}$
$= \{s : \text{Theme}(s) = \neg\text{Houdini} \sqcup ... \wedge \text{vanish}(s)\}\cup$
$\{s : \text{Theme}(s) = \neg\text{Houdini} \sqcup ... \wedge \text{turned-into-a-dove}(s)\}$
$= \{s : s \in [\![\textsf{No magician vanished}]\!]\vee$
$s \in [\![\textsf{No magician turned into a dove}]\!]\}$ Ms

This is not the correct truthmaking condition. Never mind that it is highly unclear what it means for a state to have a fusion of negative individuals as its Theme. The main issue is one of relative scope, analogous to the problem described for the Disjunctive Predication principle. We want the negation contributed by each negative individual, ¬Houdini, ¬Presto, ..., contributed by *no magician* to take scope above the disjunctive VP, which would result in a truthmaker for (86-a) being a state of Houdini neither vanishing nor turning into a dove fused with similar negative states for all the other magicians. However, the composition with Bind fails to deliver this result.

It is tempting to think that negative individuals must saturate predicates in a similar manner to positive individuals. If we view all the elements in our entity domain as representing items within our natural language ontology, then both positive and negative individuals are things complete in themselves, which are presumably capable of combining with unsaturated meanings through Functional Application, Bind, or other compositional methods. However, this assumption of 'compositional parity' is not really forced upon us, and I am inclined to reject it. As discussed earlier, we need not think of negative entities as corresponding to things in our ontology that are available for saturating meanings. Instead, according to the non-metaphysical cognitive perspective set forth in that section, speakers employ polarized data structures in their semantic processing to facilitate the smooth passing of negation through a compositional derivation. With this perspective in mind, I would like to introduce a new compositional mechanism that does just that.

The new mechanism, which I call 'Polar Bind', is designed to replace and upgrade the functionality of Bind.[15] When given a menu type argument and a functional argument that can operate on the items on a menu, Polar Bind processes each item on its menu argument separately

[15]Thanks to Matthew Mandelkern for suggesting this name.

with the functional argument and then combines the results using set union, like plain Bind. However, what sets Polar Bind apart from Bind is how it handles each element x on the menu when processing it with the function. With plain Bind, this was simply Functional Application. In contrast, Polar Bind takes into account the polarity of the parts of x. The non-null maximal positive part of x in $\mathrm{PART}^{+}(x)$, if there is one, is combined with the functional argument by Functional Application. On the other hand, non-null maximal negative parts of x in $\mathrm{PART}^{-}(x)$, if any, are processed by first applying the functional argument to their positive orthogonal counterparts and then applying the NEG function to the result. Finally, the menus generated from these individual computations are pairwise fused together (though if the computation involving either $\mathrm{PART}^{+}(x)$ or $\mathrm{PART}^{-}(x)$ returns the empty set, it is simply disregarded rather than leading to a crash).

To formulate Polar Bind, we require both the pointwise fusion operation (87), as well as this slight modification of the operation (only the binary case is needed):

(87) Where X and Y are Ma-type sets of entities or states,

$$X \uplus Y := \begin{cases} X \sqcup Y & \text{if } X \neq \emptyset \wedge Y \neq \emptyset \\ X & \text{if } X \neq \emptyset \wedge Y = \emptyset \quad (\text{whereas } X \sqcup Y = \emptyset) \\ Y & \text{if } X = \emptyset \wedge Y \neq \emptyset \quad (\text{whereas } X \sqcup Y = \emptyset) \\ \emptyset & \text{if } X = \emptyset \wedge Y = \emptyset \end{cases}$$

Bind can now be upgraded as follows:

(88) **Polar Bind**

$$\ggg \; :: \; \mathsf{Ma} \rightarrow (\mathsf{a} \rightarrow \mathsf{Mb}) \rightarrow \mathsf{Mb}$$
$$\ggg \; := \; \lambda X \lambda f. \bigcup_{x \in X} (\dot{\bigsqcup}_{y \in \mathrm{PART}^{+}(x)} f(y) \uplus \dot{\bigsqcup}_{y \in \mathrm{PART}^{-}(x)} \mathrm{NEG}(f(\neg y)))$$

Polar Bind simplifies to Bind when its Ma-type argument includes only non-null positive elements because, for any positive element x, $\mathrm{PART}^{+}(x) = \{x\}$ and $\mathrm{PART}^{-}(x) = \emptyset$:

(89)
$$\begin{aligned} X \ggg f &= \bigcup_{x \in X} \dot{\bigsqcup}_{y \in \{x\}} f(y) \\ &= \bigcup_{x \in X} f(x) \end{aligned}$$

When the Ma-type argument consists exclusively of non-null negative elements and fusions of these negative elements, Polar Bind also simplifies to the following streamlined form:

(90)
$$X \ggg f = \bigcup_{x \in X} \dot{\bigsqcup}_{y \in \mathrm{PART}^{-}(x)} \mathrm{NEG}(f(\neg y))$$

(91) a. No magician vanished.

b. $$\llbracket[[_{\mathrm{NegP}}\mathsf{not}[_{\mathrm{DP}}\mathsf{some}_{s_0}(\mathsf{magician})]][\lambda e[_{\mathrm{TP}}[\mathsf{[Th]}e][_{\mathrm{V}}\mathsf{vanished}]]]]\rrbracket$$
$$= \llbracket\mathsf{not}(\mathsf{some}_{s_0}(\mathsf{magician}))\rrbracket \ggg \lambda e.(\llbracket\mathsf{[Th]}\rrbracket(e) \cap \llbracket\mathsf{vanish}\rrbracket)$$
$$= \dot{\sqcup}_{e\in\{\neg\mathrm{Houdini},\neg\mathrm{Presto},...\}}\mathrm{NEG}(\{s : \mathrm{Theme}(s) = \neg e \wedge \mathrm{vanish}(s)\})$$
$$= \mathrm{NEG}(\{s : \mathrm{Theme}(s) = \mathrm{Houdini} \wedge \mathrm{vanish}(s)\})\dot{\sqcup}$$
$$\mathrm{NEG}(\{s : \mathrm{Theme}(s) = \mathrm{Presto} \wedge \mathrm{vanish}(s)\})\dot{\sqcup}...$$
$$= \{\bigsqcup\{\neg s : s \in \llbracket\mathsf{Houdini\ vanished}\rrbracket \vee s \in \llbracket\mathsf{Presto\ vanished}\rrbracket \vee ...\}\} \quad \mathsf{Ms}$$

This is the same truthmaking condition as the one we derived earlier in the computation (81) using sentential negation. I leave it to the reader to verify that we also obtain appropriate truthmakers for *No magician vanished or turned into a dove*, where the negation contributed by each of the anti-magicians now scopes over the VP disjunction, unlike in the earlier problematic computation (86).

The Polar Bind method ($\ggg$) can also be used to integrate predicates with hybrid elements that consist of both non-null positive and negative parts. In the upcoming section, I will demonstrate how this is precisely what we need to provide a satisfying collective treatment of coordination structures involving both upward and non-upward entailing constituents.

8 Cracking the hard nuts

As discussed in Section 2, the collective theory proposed by Heycock & Zamparelli (2005) does not adequately extend to coordination structures with negative quantifiers and other non-upward entailing DPs, leading researchers like Champollion (2016) in his earlier work to take refuge in the intersective theory (although Champollion has since aligned with the collective camp in his collaboration with Timothée Bernard). At this point, we are equipped to offer an implementation of the collective theory that yields more favorable results for these challenging cases.

Recall example (7), repeated below as (92):

(92) Agrippina and nobody else danced.

The subject *Agrippina and nobody else* can now be taken to signify the singleton set containing the hybrid entity consisting of Agrippina and the orthogonal counterparts of everybody else. When composed with Polar Bind, each individual part of this sum contributes a distinct part of a truthmaker for the sentence. Agrippina determines states of Agrippina dancing, anti-Caesar determines the state of Caesar not dancing (i.e., the fusion of the orthogonal counterparts of all the states where he dances), anti-Brutus determines the state of Brutus not dancing (i.e., the fusion of the orthogonal counterparts of all the states where he dances), and so forth. A truthmaker for (92) is the fusion of states of each kind:

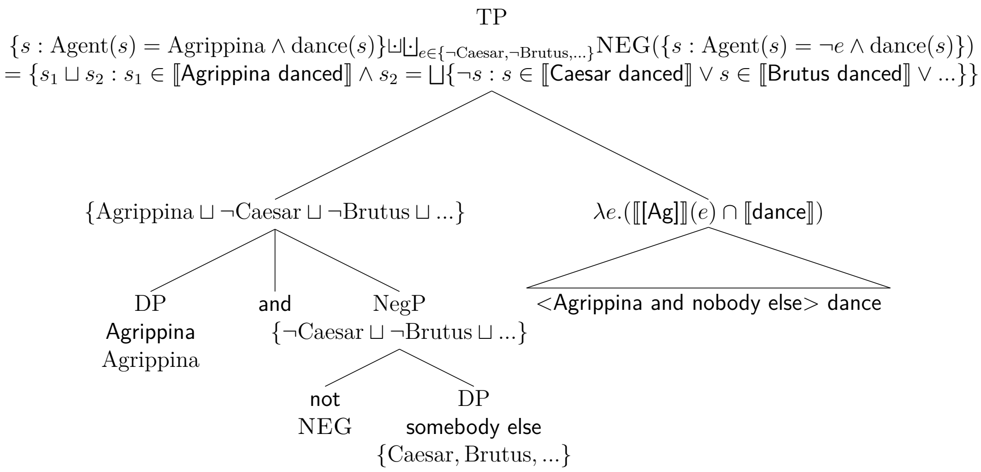

Figure 5: Derivation of Agrippina and nobody else danced

Adversative constructions receive a similar collective analysis. Take (93), for instance:

(93) Hermia but not Helena danced.

A truthmaker for this counterexpectational *but* example consists of a state of Helena dancing fused with the state of Hermia not dancing:

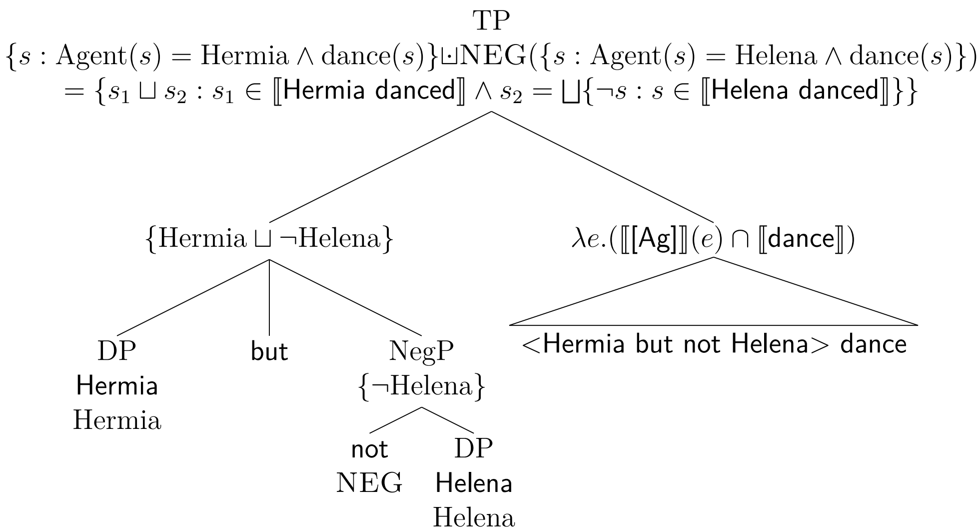

Figure 6: Derivation of Hermia but not Helena danced

And here is an example involving corrective *but*, the analysis of which I informally discussed at the end of Section 2:

(94) Not a mathematician but a physicist discovered the neutron. (Vicente 2010; Toosarvandani 2013)

I assume that the indefinite article *a* has the same alternative semantics as *some* when combined with a singular noun phrase, and I interpret *not a mathematician but a physicist* as a subclausal coordination that denotes a menu of hybrid entities each of which consists of a single physicist fused with all the anti-mathematicians. Through composition with Polar Bind, we obtain truthmakers for this example that consist of a state of a specific physicist discovering the neutron fused with negative states of the mathematicians failing to discover it:

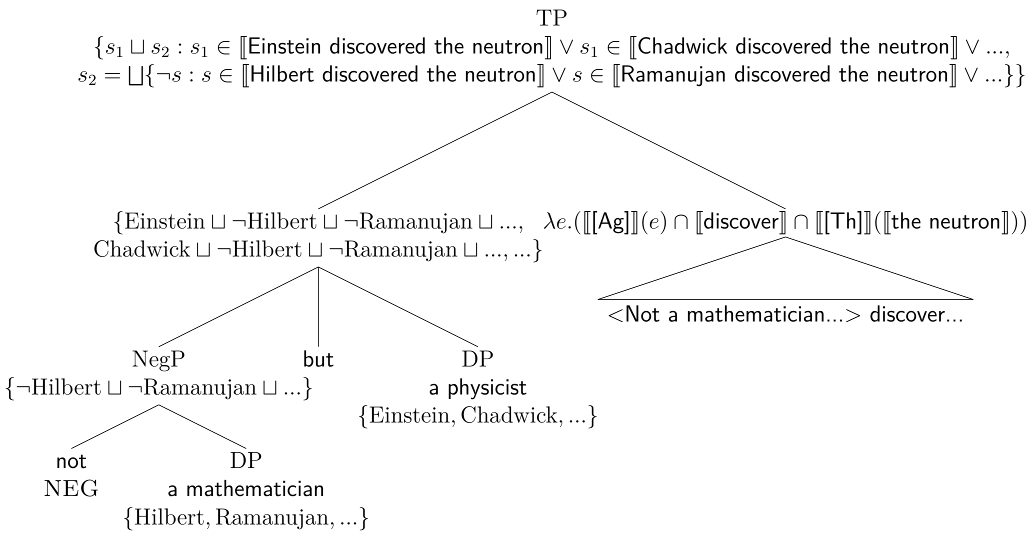

Figure 7: Derivation of Not a mathematician but a physicist discovered the neutron

This analysis assumes a syntactic parse of (94) that involves constituent negation. However, readers can verify that under an alternative Conjunction Reduction syntax with sentential negation, my proposed semantics would still yield the same truthmakers.

Having negative individuals as a resource also affords new menu type treatments of coordinations involving non-upward entailing numerical phrases in examples like the following:

(95) a. John and between one and three women smiled.
b. John and an odd number of women smiled.

The proposal to interpret numerical determiner phrases in the type of sets of entities, rather than in the type of generalized quantifiers, has already been developed in several places (Bartsch 1973; Verkuyl

1981; Link 1987; Krifka 1999; Landman 2004; among others). Following broadly what Landman (2004) calls the 'adjectival theory', we might assume, for instance, that a DP occurrence of *two women* in argument or predicate position combines with an empty determiner position ∅ and denotes a set comprising pluralities of two women (compositional details are set aside here; see pp. 12-15 for Landman's own proposal on the fine-grained structure of numerical noun phrases in terms of a number, numerical relation, and measure):

(96) $[\![\varnothing_{s^*}(\text{two women})]\!] \quad = \quad \{\text{w1} \sqcup \text{w2}, \text{w2} \sqcup \text{w3}, ...\}$ Me

Similarly, we can interpret ∅ *at least two women* as contributing an alternative set whose members are pluralities consisting of two or more women:

(97) $[\![\varnothing_{s^*}(\text{at least two women})]\!] \quad = \quad \{\text{w1} \sqcup \text{w2}, ..., \text{w1} \sqcup \text{w2} \sqcup \text{w3}, ...\}$ Me

These denotations integrate nicely into my truthmaker semantics. However, we run into immediate challenges when dealing with examples like those in (95) involving non-upward entailing numerical phrases. If, for example, we interpret ∅ *between one and three women* as denoting an alternative set of pluralities that consist of between one and three women, then we seem to derive incorrect truthmakers for (95-a): a possible truthmaker for this sentence would be, say, a state of John, Beatrice, and Claribel singing, which could be part of a larger state of John and four or more women smiling, a state which falsifies (95-a).

To address this issue, I tentatively propose an alternative interpretation of ∅ *between one and three women* as denoting an alternative set of hybrid entities, where each entity in this set is composed of a positive plurality of between one and three women fused with the negative orthogonal counterparts of every plurality of four or more women (this interpretation can be derived by parsing ∅ *between one and three women* as ∅ *at least one woman but not* ∅ *at least four women* and then filtering out any 'inconsistent' entities in the denotation that have non-null orthogonal parts):

(98)
$$[\![\varnothing_{s^*}(\text{between one and three women})]\!]$$
$$= \left\{ \begin{array}{l} \text{w1} \sqcup \neg(\text{w1} \sqcup \text{w2} \sqcup \text{w3} \sqcup \text{w4}) \sqcup ..., \\ \text{w1} \sqcup \text{w2} \sqcup \neg(\text{w1} \sqcup \text{w2} \sqcup \text{w3} \sqcup \text{w4}) \sqcup ..., \\ \text{w1} \sqcup \text{w2} \sqcup \text{w3} \sqcup \neg(\text{w1} \sqcup \text{w2} \sqcup \text{w3} \sqcup \text{w4}) \sqcup ..., ... \end{array} \right\} \quad \text{Me}$$

Conjoining with *John* tacks John onto each hybrid entity on the menu:

(99) $$[\![\textsf{John and } \varnothing_{s^*}\textsf{(between one and three women)}]\!] = \left\{\begin{array}{l} \text{John} \sqcup \text{w1} \sqcup \neg(\text{w1} \sqcup \text{w2} \sqcup \text{w3} \sqcup \text{w4}) \sqcup ..., \\ \text{John} \sqcup \text{w1} \sqcup \text{w2} \sqcup \neg(\text{w1} \sqcup \text{w2} \sqcup \text{w3} \sqcup \text{w4}) \sqcup ..., \\ \text{John} \sqcup \text{w1} \sqcup \text{w2} \sqcup \text{w3} \sqcup \neg(\text{w1} \sqcup \text{w2} \sqcup \text{w3} \sqcup \text{w4}) \sqcup ..., ... \end{array}\right\} \quad \textsf{Me}$$

Applying Polar Bind yields appropriate truthmakers:

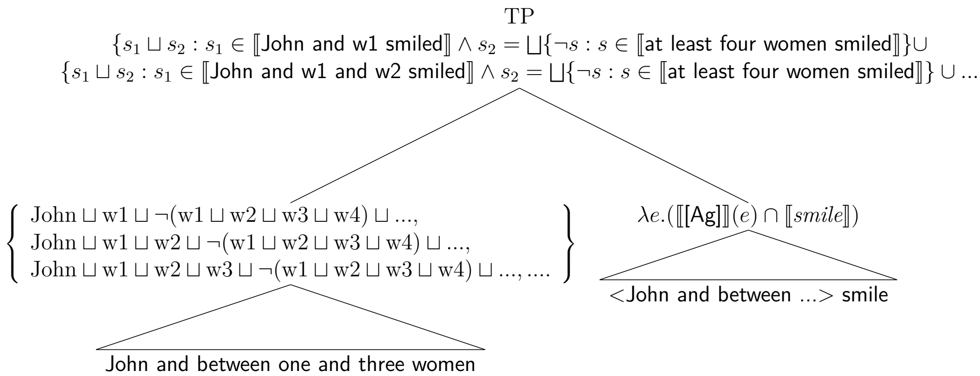

Figure 8: Derivation of John and between one and three women smiled

Notice that the negative entities in the denotation of the subject *John and between one and three women* contribute the required maximalization effect, ensuring that the one, two, or three women featuring in a truthmaking state are the only women who smiled.

I describe my proposal as ‘tentative’ because modified numerals present several challenging interpretative puzzles that I have yet to fully address. One of the most difficult challenges involves providing a compositional treatment of cumulative readings for sentences with multiple non-upward entailing modified numerals, as exemplified by sentence (100) (Krifka 1999; Brasoveanu 2013; this is sometimes referred to as “van Benthem’s puzzle” after van Benthem 1986):

(100) Between one and three women saw between two and four movies.

On the cumulative reading of this sentence, there was an event of women watching movies, where the maximum number of women who saw a movie was between one and three, and the maximal number of movies seen by a woman was between two and four. This differs from the surface-scope distributive reading, where one, two, or three women each saw two, three, or four movies, potentially resulting in a total number of movies seen exceeding four. A satisfying comprehensive account of modified numerals must allow for the compositional derivation of cumulative readings. However, my semantics currently does not provide for this. Therefore, while I believe my treatment of non-upward entailing modified numerals in terms of negative entities is a promising approach that deserves further exploration, it remains incomplete.

9 Excluding states

We have reached that point in a paper where I am expected to consider competing proposals from the literature and show how my theory improves on everything that came before. The most closely related competitors are the unilateral truthmaker semantic treatments of negation developed by Kit Fine and by Lucas Champollion and Timothée Bernard that rely on an exclusion relation between states (Fine 2017a; Champollion & Bernard 2024). In this section, however, I prefer to take a more conciliatory approach: rather than arguing against these exclusion-based theories, I will briefly demonstrate how an exclusion relation can be incorporated into my own theory.

Up until now, my semantics has been non-modal. The distinction between positive and negative states, like the distinction between positive and negative entities, is not intended to capture an underlying modal notion of *incompatibility*, and at no point in this article have I made reference to the notion of a *possible state*. With the introduction of an exclusion relation, however, I am now bringing a modal element into my semantics. The semantic theories of Fine and Champollion & Bernard can be situated within a broader tradition that can be traced back to the work of Goldblatt (1974) and of Dunn (1993), who explicitly analyze negation as a modal operator in terms of an incompatibility relation between situations (see Beall & Restall 2006; Berto 2015 for some more recent discussion). Exclusion can be regarded as an exactification of this inexact incompatibility semantics.

Fine and Champollion & Bernard impose different constraints on the exclusion relation in their respective theories. Fine's exclusion relation is not symmetric, meaning that one state can exclude another, but not necessarily vice versa. For example, Socrates being Greek excludes him being a Roman philosopher, but the reverse does not hold, since being a philosopher does no work in excluding Socrates from being Greek. On the other hand, Champollion & Bernard employ a symmetric exclusion relation. In their approach, Socrates being Greek excludes only his being Roman, and vice versa (though if being a philosopher excludes and is excluded by being, say, a wrestler, then Socrates being a Greek wrestler would exclude and be excluded by his being a Roman philosopher, given a further "cumulativity" axiom that they impose on exclusion).

While the exclusion relations in these theories differ in certain other respects as well, the choice between them does not significantly impact the present discussion. In this section, I adopt a symmetric exclusion relation inspired by Champollion & Bernard's approach because I believe it aligns better with the exact framework, although the analysis could have been developed using Fine's approach instead.

I assume that the symmetric exclusion relation on the state space, denoted as $\perp$, holds only between positive states. This maintains a clear distinction between the modal structure introduced by exclusion and the polarity marking of states as positive or negative, which affects only their compositional processing. When $s \perp t$, I say that *s excludes t*, or that *t is excluded by s*. With the exclusion relation in place, we can further define the notion of a state 'precluding' another state when the former excludes one of the parts of the latter:

(101) s **precludes** t just in case $s \perp t'$ for some $t' \leqslant t$.

Champollion & Bernard define event negation in terms of preclusion:

(102) **Champollion & Bernard's NEG function**

$$[\![\mathsf{not}]\!] \quad = \quad \lambda S. \dot{\bigsqcup}_{s \in S}\{t : t \text{ precludes } s\} \qquad \mathsf{Ms} \rightarrow \mathsf{Ms}$$

This is quite close to my own NEG function (limited to state arguments), except that the precluders of a state have replaced the members of its orthogonality set. Consequently, a truthmaker for a simple negative sentence like *Socrates wasn't Roman* is now a fusion of precluders for each of the truthmakers for *Socrates was Roman*, instead of a fusion of the negative orthogonal counterparts of these truthmakers.

To obtain truthmaking conditions similar to those of Champollion & Bernard for such negative sentences, we can substitute precluders for negative states at the end of a semantic derivation. The following 'exclusionary transformation' of one menu of states into another accomplishes this effect:

(103) **Exclusionary transform**

$$\mathcal{T}_{\perp} \quad :: \quad \mathsf{Ss} \rightarrow \mathsf{Ss}$$
$$\mathcal{T}_{\perp} \quad := \quad \lambda S. \bigcup_{s \in S}(\mathrm{PART}^{+}(s) \uplus \dot{\bigsqcup}_{\neg t \in \mathrm{PART}^{-}(s)}\{t' : t' \text{ precludes } t\})$$

To see this at work, consider the following sentence, which I assume, for simplicity, to have as its sole truthmaker the fusion of a state SR of Socrates being Roman, a state SP of Socrates being a philosopher, and the negative orthogonal counterpart of the state MR$\sqcup$MP of Milo being a Roman philosopher:

(104) a. Socrates was a Roman philosopher but Milo was not.

b. $[\![[[_{\text{TP}}\textsf{Socrates was a Roman philosopher}]\ \textsf{but}\ [_{\text{NegP}}\textsf{not}[_{\text{TP}}\textsf{Milo was a Roman philosopher}]]]\!]$

$$= \{\mathrm{SR} \sqcup \mathrm{SP}\} \dot{\sqcup} \mathrm{NEG}(\{\mathrm{MR} \sqcup \mathrm{MP}\})$$
$$= \{\mathrm{SR} \sqcup \mathrm{SP} \sqcup \neg(\mathrm{MR} \sqcup \mathrm{MP})\} \qquad \textsf{Ms}$$

Applying the exclusionary transformation replaces the negative part $\neg(\mathrm{MR} \sqcup \mathrm{MP})$ with any of the precluders of the negated state $\mathrm{MR} \sqcup \mathrm{MP}$, such as a state MG of Milo being Greek, a state MW of Milo being a wrestler, a state $\mathrm{MG} \sqcup \mathrm{MW}$ of Milo being a Greek wrestler, and so on:

(105) $$\mathcal{T}_{\perp}((104\text{-b})) = \{\mathrm{SR} \sqcup \mathrm{SP} \sqcup \mathrm{MG}, \mathrm{SR} \sqcup \mathrm{SP} \sqcup \mathrm{MW}, ...\} \qquad \textsf{Ms}$$

I regard it a mere terminological dispute whether to call the states computed in (104-b) or those computed in (105) *the* truthmakers of (104-a). In a framework that incorporates both negative states and excluders, we might refer to the former as the ‘compositional truthmakers’ and the latter as the ‘exclusionary truthmakers’.

10 Conclusion

In one sense, this article serves as an extended argument for the claim that conjunctions involving non-upward entailing determiner phrases do not decisively undermine the collective theory. If English speakers employ entity denotations with varying polarities and incorporate this polarity marking into semantic composition, they may consistently and uniformly interpret conjunction as a means to form pluralities. A broader aim of this article, however, has been to explore a nonstandard perspective on logical expressions in natural language organized around the metaphor of a ‘menu’, which integrates recent work in truthmaker semantics with related traditions in linguistics. Although my ‘menu semantics’ may seem fanciful to some due to its colorful cast of negative characters, I have aimed to demonstrate that this approach is coherent and deserving of further investigation.

Looking ahead, one immediate and pressing task is to explore the extent to which the non-Montagovian entity-style treatment of ‘quantifiers’ introduced in Section 6 can be extended beyond the traditional cases of *every*, *some*, and *no*. In particular, it will be interesting to investigate how this treatment can be reconciled with existing non-GQT approaches to other quantifiers, such as Martin Hackl’s influential work on comparative and proportional quantifiers like *most*, *more than half*, and *few* (Hackl 2001, 2009). While I provided tentative semantic proposals for certain numerical DPs like *two women*, *at least two women*, and *between one and three women* in Section 8, it became apparent that more work is needed.

Broadening our perspective beyond the fine-grained details of semantic composition, the version of truthmaker semantics developed in this article should also be situated within a systematic account of meaningful communication, or a ‘truthmaker pragmatics’. This theory of discourse dynamics would explore how truthmakers interact with pragmatic aspects of language use and how they contribute to the overall meaning and interpretation of utterances in context.

In the classic Stalnakerian model of discourse oriented around the notion of *common ground*, a conversation takes place relative to a set of possible worlds, or a *context set*, compatible with what the speakers presuppose to be the case, and assertions and other informational contributions serve to contract or thin this space of live options by excluding worlds incompatible with accepted content (Stalnaker 1978, 2002, 2014). The truthmaker approach to semantic content affords a somewhat different perspective, where speakers are ‘constructing’ a world, state by state. We can still think of the members of a context set as ‘live options’ in the discourse, though these can now be parts of worlds rather than worlds themselves:

(106) A **context** $c \subseteq S$ is a nonempty set of states.

The default setting is a the *null context*, which contains only the null state. Making an assertion with a sentence S updates a context c by pairwise fusing the states in c with the verifiers for the uttered sentence:

(107) The **assertive update** of c by S is $c + S := c \sqcup [\![S]\!]$.

The traditional Stalnakerian picture of assertion as narrowing down a set of maximally specific possible worlds can be recovered within this truthmaker dynamics by considering the set of possible worlds that have members of a context c as parts. However, the extra structure offered by the truthmaker framework, in which speakers build up a menu of ways for what has been said or presupposed in a discourse to be made true, opens up new possibilities for analyzing pragmatic phenomena.

A Polarized entity and state spaces

In this appendix, I present a method to construct the entity and state spaces required for my menu semantics, inspired by the construction of free objects in universal algebra (Birkhoff 1940; Burris & Sankappanavar 1981). We can assume that both spaces are built up from a nonempty set X of generators, the basic individuals and states. This set X is extended 'through syntax' by applying the symbols $\neg$ and $\bigsqcup$ to elements and sets of elements in the following inductive process (similar to the generation of a *term algebra*, except that my construction can halt after just a few steps instead of continuing ad infinitum):

$$\begin{array}{lcl} X_0 & = & X \\ X_1 & = & X_0 \cup \{\bigsqcup X' : X' \subseteq X_0\} \\ X_2 & = & X_1 \cup \{\neg x : x \in X_1\} \\ X_3 & = & X_2 \cup \{\bigsqcup X' : X' \subseteq X_2\} \end{array}$$

Suppose for example that X_0 is a set of individuals $\{\text{Alfonso}, \text{Claribel}, ...\}$. The first pluralization step creates new elements obtained by attaching $\bigsqcup$ to each subset of X_0:

$$X_1 \quad = \quad \{\text{Alfonso}, \text{Claribel}, ..., \bigsqcup\emptyset, \bigsqcup\{\text{Alfonso}\}, ..., \bigsqcup\{\text{Alfonso}, \text{Claribel}\}, ...\}$$

The second polarization step adds negative counterparts of the elements in X_1:

$$\begin{array}{lcl} X_2 & = & \{\text{Alfonso}, \text{Claribel}, ..., \bigsqcup\emptyset, \bigsqcup\{\text{Alfonso}\}, ..., \bigsqcup\{\text{Alfonso}, \text{Claribel}\}, ..., \\ & & \neg\text{Alfonso}, \neg\text{Claribel}, ..., \neg\bigsqcup\emptyset, \neg\bigsqcup\{\text{Alfonso}\}, ..., \neg\bigsqcup\{\text{Alfonso}, \text{Claribel}\}, ...\} \end{array}$$

The final pluralization step again creates new elements by applying $\bigsqcup$ to subsets of X_2:

$$\begin{array}{lcl} X_3 & = & \{\text{Alfonso}, \text{Claribel}, ..., \bigsqcup\emptyset, \bigsqcup\{\text{Alfonso}\}, ..., \bigsqcup\{\text{Alfonso}, \text{Claribel}\}, ..., \\ & & \neg\text{Alfonso}, \neg\text{Claribel}, ..., \neg\bigsqcup\emptyset, \neg\bigsqcup\{\text{Alfonso}\}, ..., \neg\bigsqcup\{\text{Alfonso}, \text{Claribel}\}, ..., \\ & & \bigsqcup\{\bigsqcup\emptyset\}, \bigsqcup\{\bigsqcup\{\text{Alfonso}\}\}, ..., \bigsqcup\{\text{Alfonso}, \neg\bigsqcup\{\text{Alfonso}, \text{Claribel}\}\}, ...\} \end{array}$$

Given how my semantics for negation works, no further pluralization or polarization steps are needed.

Now, X_3 is not exactly what we want. Since applying the symbols $\neg$ and $\bigsqcup$ syntactically always creates new elements, the construction generates distinctions between various elements that should be identified. To ensure that $\bigsqcup$ serves as a generalized fusion operation, we should avoid distinguishing between elements like Alfonso, $\bigsqcup\{\text{Alfonso}\}$, $\bigsqcup\{\bigsqcup\{\text{Alfonso}\}\}$, and so on, as well as between elements like $\bigsqcup\{\text{Alfonso}, \text{Berta}, \text{Claribel}\}$ and $\bigsqcup\{\text{Alfonso}, \bigsqcup\{\text{Berta}, \text{Claribel}\}\}$. To ensure that $\neg$ is an involution, we should not distinguish between elements like Alfonso and $\neg\neg$Alfonso either. And we want a bottom element, the null entity, to serve as its own negative counterpart.

To remove undesired distinctions, we can impose an equivalence relation $\equiv$ on X_3 and then work with the resulting *quotient algebra* $\langle X_3/{\equiv}, \bigsqcup, \neg\rangle$, whose universe $X_3/{\equiv}$ is the set of equivalence classes determined by $\equiv$, which merges together elements in X_3 that should be identified, and whose operations $\bigsqcup$ and $\neg$ are interpreted in the expected manner over this universe. From an algebraic perspective, a semilattice is standardly characterized as a structure with one associative, commutative, and idempotent binary operation. Since the generalized join operation $\bigsqcup$ applies to a set of elements and is independent of any possible ordering of this set, commutativity comes for free, and we need only the following general idempotence and associativity conditions to ensure that the quotient algebra is a complete join semilattice under the ordering $x \leq y := \bigsqcup\{x, y\} = y$:

Idempotence $\bigsqcup\{x\} \equiv x$ for all $x \in X_3$.

Associativity Given any nonempty family $\mathcal{F}$ of subsets of X_3, $\bigsqcup\{\bigsqcup X' : X' \in \mathcal{F}\} \equiv \bigsqcup \bigcup \mathcal{F}$.

The next two conditions ensure that the negation operation $\neg$ is an involution and is reflexive on the null element:

Involutiveness $\neg\neg x \equiv x$ for all $x \in X_3$.

Nullity $\bigsqcup \emptyset \equiv \neg \bigsqcup \emptyset$.

And one final bit of bookkeeping: the equivalence $\equiv$ must be a 'congruence relation' (in a suitably generalized sense) with respect to the fusion and negation operations, meaning that $\neg x \equiv \neg y$ whenever $x \equiv y$, and $\bigsqcup X \equiv \bigsqcup Y$ whenever $X \equiv Y$ (where $X \equiv Y$ if and only if for each $x \in X$ there is a $y \in Y$ such that $x \equiv y$, and for each $y \in Y$ there is an $x \in X$ such that $x \equiv y$). There are likely to be many equivalence relations satisfying the above conditions, among them being the universal relation that treats all elements as equivalent, but we can take $\equiv$ to be the finest equivalence relation that meets these conditions.

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